Rolling tachyon in Anti-de Sitter space-time

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ABSTRACT: We study the decay of the unstable D-particle in three-dimensional anti-de Sitter space-time using worldsheet boundary conformal field theory methods. We test the open string completeness conjecture in a background for which the phase space available is only field-theoretic. This could present a serious challenge to the claim. We compute the emission of closed strings in the $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4$ background from the knowledge of the exact corresponding boundary state we construct. We show that the energy stored in the brane is mainly converted into very excited long strings. The energy stored in short strings and in open string pair production is much smaller and finite for any value of the string coupling. We find no "missing energy" problem. We compare our results to those obtained for a decay in flat space-time and to a background in the presence of a linear dilaton. Some remarks on holographic aspects of the problem are made.

KEYWORDS: Tachyon Condensation, D-branes, AdS-CFT Correspondence.

Contents

1. Introduction and summary

Understanding the physics of time-dependent backgrounds is one of the the greatest challenges in string theory. It is important for the understanding of basic issues in quantum gravity and for obtaining some hints of stringy effects in primordial cosmology. The current technology, based on a topological expansion of string worldsheets in a first quantized framework, leaves many problems unsolved, such as understanding the analytical continuation to Lorentzian signature and backreaction that arises in time-dependent space-times. Some exact two-dimensional conformal field theories with a time-dependent target space-time were analyzed [1–5]. It is not easy to extract physical information from them, in

particular due to the presence of singularities in the correlators in some cases [6−8], that may signal a large back-reaction on the geometry [9, 10] and the breakdown of perturbation theory (see however [11]).

Time-dependent backgrounds also play an important role in the search for new string vacua in the string background landscape. Detecting and following infrared instabilities in space-time and on the worldsheet has been a tool for such investigations. The more risky expeditions were those following the trajectories of closed string tachyons (see for example [12]). Following open string tachyons (see [13, 14], the review [15] and references therein) and localized closed string tachyons (see for example [16, 17]) is somewhat more tractable and also of great interest.

In this paper we study some aspects of a particular time-dependent process involving the decay of unstable D-branes, described by an integrable boundary deformation of the string worldsheet [18, 19]. The main simplification is that the minimum of the tachyon potential, the closed string vacuum, is known [20]. The study of open string tachyon condensation is not only important to unveil fundamental aspects of string theory dynamics but also to attempt to build realistic cosmological scenarios in string theory [21, 22].

The tree-level computation of closed string production leads to an exponentially divergent emission of highly massive non-relativistic excited strings [23, 24]. This divergence is unphysical for any non-zero value of g_s (because the initial energy available is finite and equal to the D-brane mass), and indicates in that case inherent difficulties in a perturbative analysis. Considerations following from the space-time effective action also suggest that the remnant of the process is a pressure-less "tachyon dust", whose nature was studied in [18, 25] and characterized there as somewhat mysterious. The main issue addressed was if the open string description in terms of tachyon dust can be reformulated in terms of a closed string description. If it would turn out that the closed string component of the decay can't account for all the energy available in the D-brane before its decay than a "missing energy" puzzle would emerge. It would not be clear what type of matter constitutes the reminder of the decay products of the D-brane. The formulation of the question is actually more involved. In the tree-level approximation the mass of the decaying D-brane is infinite. If the tree-level approximation for the energy deposited in closed strings is finite and does not depend on g_s (UV finite) then a puzzle emerges, because the first estimate of the finite energy density of the brane would be that it is of order $1/g_s$.

In this paper we examine the "missing energy" issue in the circumstances where that phase space is minimized, that is the case of AdS$_3$ where the actual entropy is not Hagedorn like but only field-theoretic. We mainly consider the decay of an unstable D0-brane in this background. As the amount of closed string radiation is also proportional to the phase space available for the closed strings, one may expect this case to be a most severe challenge to the *open string completeness* conjecture of Sen [26], which states that the open string field theory description of the tachyon decay (approximated by the tachyon effective action) captures all the physics of the decay process, in particular that it gives a "holographically dual" description of closed string radiation, even when closed string perturbation theory breaks down. We find that in fact the outcome of the computation will be qualitatively similar to brane decay in flat space-time. In the tree approximation the energy stored in

the closed strings is much too high and one must identify a mechanism to decrease the amount of energy carried by the closed strings. The open string picture on the other hand remains rather simple.

String theory in AdS$_3$ with NS-NS fluxes is described by the (universal cover of the) SL$(2, \mathbb{R})$ WZW model [27]; it is a solvable conformal field theory. The study of the associated boundary conformal field theory allowed to construct several types of SL$(2, \mathbb{R})$-preserving and symmetry-breaking D-branes, first in the related Euclidean H$_3^+$ model [28, 29] and then in the case of AdS$_3$ with Lorentzian signature [30]. Among the symmetry-breaking D-branes is a D-particle sitting at the origin of global coordinates [31]. In this work we consider the non-BPS D0-brane in type IIB superstrings on AdS$_3 \times$ S$^3 \times$ T^4 built out of it. We study its spectrum of fluctuations (open string modes) that contains a tachyon.

The presence of this open string tachyon, and the particular structure of the boundary state associated with this D-particle, allows one to find the exact boundary state describing the time-dependent development of this instability. The boundary deformation that is turned on has the form (in the bosonic case) $\oint_{\partial \Sigma} d\ell \, \exp(X^0(\ell)/\sqrt{\alpha'})$, i.e. the same as one would obtain for an unstable D-brane in flat space-time, although the boundary state is different.

There are two (related) important differences though between the flat space-time case and the current analysis in AdS. First, the scaling of space-time energy with the oscillator number of the string is quite different (i.e. $E_{\text{AdS}} \sim N$ vs. $E_{\text{FLAT}} \sim \sqrt{N}$), meaning that the number of string states for a given energy is smaller. As a consequence, one may envisage that the production of very excited closed strings could be highly suppressed relative to flat space-time.

A second striking difference with the rolling tachyon in flat space-time is the existence of "long strings", i.e. macroscopic circular strings whose radius grows linearly w.r.t. AdS global time, while having a finite energy because their (infinite) mass is balanced by the coupling to the NS-NS two-form [32, 33]. We find that they are copiously produced by the brane decay, in such a way that an unrestricted sum over the winding sectors would give a divergent energy for the closed string radiation. This is obtained by computing the annulus amplitude for the rolling tachyon boundary state, whose imaginary part gives the mean number of emitted closed strings [34].

Non-perturbative physics does provide a cutoff on the magnitude of the winding number w (which has to be smaller than the number of background fundamental strings) which manages to regularize both the mean number of emitted closed strings and all moments of the emitted energy. However it is not stringent enough to give a correct estimate of the radiated energy. The finite energy obtained is of order $1/g_s^2$ while the total energy stored in the brane is only of order $1/g_s$. This finite result consists of important contributions of long string states whose energy is larger than $1/g_s$. Such contributions, although finite, are outside the realm of a reliable perturbative computations. As the energy stored in closed strings is larger than the available energy one should try to understand how the actual amount of energy is of order $1/g_s$. If for some reason only closed strings with energies smaller than $1/g_s$, i.e. strings whose energy we can reliably calculate, would contribute significantly to the computation of the mean radiated energy, a fully satisfactory result would

be obtained.

Another way to regularize the high-energy divergence of closed strings radiation that was advocated in the past is to consider non-critical string theories [34]. Such string backgrounds have a higher Hagedorn temperature compared to critical strings in flat space-time, therefore their ultraviolet behavior is somewhat softer. Previous experience suggests that anti-de Sitter space-time is the best possible regulator because its density of states is the same as that of a field theory. Therefore we do not expect closed string emission by a D-brane decay in linear dilaton backgrounds to be more suppressed at high energies than in AdS. Analyzing some examples of rolling tachyon in non-critical superstrings we show that closed string emission, computed at tree level, is plagued with the same divergences. Thus in both examples, AdS and linear dilaton, string theory manages cleverly to dissipate all the energy stored in the brane.

We compute the rate of open string pair creation [19] by the rolling tachyon in AdS_3. We show that it is exponentially suppressed at high energies. Therefore perturbative string theory provides a valid description of the open string side of the process. These results are in agreement with Sen's hypothesis. The effective open string description in flat space-time and AdS_3 share some similarities; the tachyon effective action – which has the same domain of validity as in flat space-time if we choose the radius of AdS to be large w.r.t. the string length – predicts similar dynamics. However, the worldsheet CFT description shows that while in the former case the mean number of emitted open strings is only power-like decreasing at high energies, in the latter example the convergence is exponential-like. The closed string side of the story seems very different. Instead of a "dust" of non-relativistic massive closed strings, one gets a "ring" of long strings with very large winding number expanding at a common constant speed.

A different route to time-dependence in string theory is to use the holographic dualities between gravitation and gauge theory, whose hallmark is the AdS/CFT correspondence [35]. For instance the decay of an unstable sphaleron in the gauge theory has been considered in [36], after the proper $AdS_5/$ CFT_4 identification of the static configuration has been found in [37]. However the field theory computations were compared to the flat space rolling tachyon solution for lack of knowledge of the $AdS_5 \times$ S^5 worldsheet theory. The example studied in this paper, D-particle decay in AdS_3, is a very close analogue of this setup with the advantage of having a better control on the AdS side of the correspondence.

By analogy with the $AdS_5 \times$ S^5 example we expect that the D-particle in AdS_3 is dual to a sort of sphaleron of the space-time CFT associated with the instanton dual to the D-instanton in anti de-Sitter. However the space-time CFT is notoriously hard to study because it is singular in the regime where the bulk theory is solvable, i.e. without Ramond-Ramond fluxes. We leave as an open problem the precise description of the D0-brane dual in the space-time theory. This would help to understand the physics of the decay at a non-perturbative level, in particular to understand better the regularization of long strings production.

The production of long strings is one of the features that render the holographic interpretation complicated, because it decreases the central charge of the space-time conformal field theory. The latter is the product of two CFTs (one free and one interacting), the

interacting one having a central charge proportional to Q_1, the number of fundamental strings that build the AdS$_3$ background. The long strings emission by the D0-brane decay decreases their number. We expect eventually that after non-perturbative effects are taken into account, in order that the energy released into closed strings radiation is of the order of the D0-brane mass, $\delta c/c = \delta Q_1/Q_1$ will be of order $1/\sqrt{Q_1}$. If the cutoff that we use turns out to be appropriate, the relative variation of the central charge is very small in the perturbative regime.

This paper is organized as follows. In section 2 we recall some aspects of string theory in AdS$_3$ and discuss the non-BPS D-particle in AdS$_3\times$ S$^3\times$ T^4. In section 3 we briefly review the rolling tachyon in flat space-time and construct the rolling tachyon boundary state in AdS$_3$. Section 4 is devoted to the analysis of closed string emission in AdS$_3\times$S$^3\times$ T^4. This is the main result of this work. We discuss also the emission of open strings. In section 5 we make some comments on the holographic interpretation of the decay.

2. D-particles and D-instantons in AdS$_3\times$ S$^3\times$ T^4

In this section we construct in detail the unstable D-particle in AdS$_3\times$ S$^3\times$ T^4. The reader interested mostly by the decay of the brane can move directly to section 3.

We will first set the stage with a short review of string theory on AdS$_3$. Then we will recall the construction of the D(-1)- and D0-branes in AdS$_3$ at the microscopic boundary CFT level and embed the latter in the AdS$_3\times$ S$^3\times$ T^4 type IIB superstring background, the example we will follow in the next sections.

2.1 Strings and D-branes in AdS$_3$

String theory on AdS$_3$, with an NS-NS 2-form flux, is described by the WZW model for (the universal cover of) SL$(2,\mathbb{R})$. In the following we will consider the supersymmetric WZW model that is used to construct superstring theories. The corresponding background fields for a supersymmetric affine $\widehat{\mathfrak{sl}}(2,\mathbb{R})$ algebra[1] at level k are:

$$\mathrm{d}s^2 = \alpha'k\left[\mathrm{d}\rho^2 + \sinh^2\rho\,\mathrm{d}\phi^2 - \cosh^2\rho\,\mathrm{d}t^2\right]$$
$$H = 2\alpha'k\cosh\rho\sinh\rho\,\mathrm{d}\rho\wedge\mathrm{d}\phi\wedge\mathrm{d}t\,, \tag{2.1}$$

using the global coordinates. The dilaton field is a constant that we choose such that the string coupling $g_s = \exp\Phi$ is small, in order to justify worldsheet CFT techniques. The central charge of this superconformal theory is $c = 9/2 + 6/k$.

The closed string spectrum splits into standard $\widehat{\mathfrak{sl}}(2,\mathbb{R})$ representations ($w = 0$) and twisted ones ($w \neq 0$) obtained by an outer automorphism called *spectral flow* [38, 33]. The weights of the primaries read, in the NS sector:

$$L_0 = -\frac{j(j-1)}{k} - wm + \frac{k}{4}w^2 \quad , \qquad \bar{L}_0 = -\frac{j(j-1)}{k} - w\bar{m} + \frac{k}{4}w^2\,, \tag{2.2}$$

where $(m,\bar{m})$ label primaries of the $\hat{\mathfrak{u}}_1$ elliptic compact sub-algebra $(J^3,\bar{J}^3)$. Space-time energy is given by $E = m + \bar{m}$ whereas the angular momentum (conjugate to ϕ) is $n =$

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[1]Containing a purely bosonic $\widehat{\mathfrak{sl}}(2,\mathbb{R})$ algebra at level $k + 2$.

$m - \bar{m}$. The $\mathfrak{sl}(2, \mathbb{R})$ representations appearing in the unitary closed string spectrum fall into two classes. The *discrete representations* with [33]

$$\frac{1}{2} < j < \frac{k+1}{2} \tag{2.3}$$

are related to string worldsheets trapped inside AdS$_3$, corresponding to classical solutions obtained by spectral flow of time-like geodesics. The *continuous representations* with $j = \frac{1}{2} + iP$, $P \in \mathbb{R}_+$ are related to closed strings whose wave-functions are delta-function normalizable. They correspond to circular long string solutions growing linearly w.r.t. global time, with asymptotic winding number w, obtained by the spectral flow of space-like geodesics.

It is convenient for many computations to decompose the WZW model in terms of the coset SL$(2, \mathbb{R})$/U(1), the Euclidean 2D black hole [12, 39, 40], and a time-like boson:

$$\mathrm{SL}(2, \mathbb{R})_k \sim \frac{\mathrm{SL}(2, \mathbb{R})/\mathrm{U}(1)|_k \times \mathrm{U}(1)_{-k}}{\mathbb{Z}}, \tag{2.4}$$

We refer the reader to appendix A and to [30] for more algebraic details.

Localized D-branes in AdS$_3$. As in any WZW model, the *symmetric* D-branes in SL$(2, \mathbb{R})$ are defined by the gluing conditions for the currents of the affine algebra, which includes a possible twist by an (outer) automorphism of the algebra preserving the gluing condition for the ($\mathcal{N} = 1$ super-)conformal algebra [41].

The D-instanton that we will be interested in is given by the trivial gluing conditions (in the open string channel) [42]:

$$J^3(z) = \bar{J}^3(\bar{z})|_{z=\bar{z}} \quad , \qquad J^\pm(z) = \bar{J}^\pm(\bar{z})|_{z=\bar{z}}. \tag{2.5}$$

It corresponds to a point-like object in AdS$_3$ space-time, localized at $\rho = t = 0$ in global coordinates. The exact construction of the boundary state in Lorentzian AdS$_3$ [30] follows from the same logic that we discussed above, starting with the D-branes in SL$(2, \mathbb{R})$/U(1) [43–46] that can be obtained either from the gauging of H_3^+ [28], the Euclidean continuation of AdS$_3$ or using the conformal bootstrap of the $\mathcal{N} = 2$ superconformal algebra with $c > 3$ [47, 48]. The one-point function for an NS-NS primary field $V^j_{m\bar{m}w}(z, \bar{z})$ in the presence of this localized "S-brane" reads [30]:

$$\langle V^j_{m\bar{m}w}(z, \bar{z}) \rangle = \frac{1}{|z - \bar{z}|^{2\Delta_{jmw}}} \frac{2i\pi}{(2k)^{3/4}} \nu_k^{1/2-j} \frac{\Gamma(j + m - \frac{kw}{2})\Gamma(j - m + \frac{kw}{2})}{\Gamma(2j-1)\Gamma(1 + \frac{2j-1}{k})} \delta_{m-\bar{m},0}, \tag{2.6}$$

where $\nu_k = \Gamma(1 - 1/k)/\Gamma(1 + 1/k)$ and Δ_{jmw} is the conformal dimension of the primary as given by eq. (2.2). It contains couplings to closed strings both in the continuous and discrete representations. The former are obtained by evaluating (2.6) for $j \in 1/2 + i\mathbb{R}_+$, while the latter are the *residues* at the poles of the one-point function for real j. In the semi-classical regime $k \to \infty$, these couplings can be found by applying the distribution $\delta(\rho)\delta(t)$ to the (delta-function) normalizable wave-functions on AdS$_3$, as in [43] for the coset model.

The open string annulus amplitude is obtained by closed/open channel duality using this one-point function[2] Still concentrating only on the NS sector, we get in Lorentzian space-time the *identity representation* of $SL(2,\mathbb{R})$[3] (with $q = \exp 2i\pi\tau$):[4]

$$Z(\tau) = \sum_{r \in \mathbb{Z}} ch_{\mathbb{I}}\begin{bmatrix} 0 \\ 0 \end{bmatrix}(r;\tau) \frac{q^{-\frac{r^2}{k}}}{\eta(\tau)} \left(\frac{\vartheta\begin{bmatrix} 0 \\ 0 \end{bmatrix}(\tau)}{\eta(\tau)} \right)^{1/2} , \qquad (2.7)$$

that we decomposed in terms of $U(1)$ characters and the "string functions" $ch_{\mathbb{I}}\begin{bmatrix} 0 \\ 0 \end{bmatrix}(r;\tau)$ of the super-coset $SL(2,\mathbb{R})/U(1)$ for the identity representation. To get a proper interpretation of this brane as a D-instanton one should consider an Euclidean target space instead, i.e. Wick rotate AdS_3 to H_3^+. We would like also to stress that, unlike in the closed string sector, the identity representation – containing in particular the vacuum state – is present in this sector of open strings attached to a localized brane.

Using the coset decomposition (2.4) one can construct *symmetry-breaking D-branes* [50] associated with the same boundary conditions in the $SL(2,\mathbb{R})/U(1)$ coset. It boils down to replacing Dirichlet boundary conditions by Neumann ones for the time-like boson, changing appropriately the overall normalization of the boundary state to satisfy the Cardy condition (i.e. such that the identity appears in the open string spectrum with coefficient one). This D-brane is interpreted geometrically as a D-particle sitting at the origin of AdS_3 [31]. The associated one-point function for an NS-NS primary is [30]:[5]

$$\langle V_{m\bar{m}w}^j \rangle = i\pi \left(\frac{2}{k} \right)^{1/4} \nu_k^{1/2-j} \; \frac{\Gamma(j + \frac{kw}{2})\Gamma(j - \frac{kw}{2})}{\Gamma(2j-1)\Gamma(1 + \frac{2j-1}{k})} \; \delta_{m-\bar{m},0} \; \delta(m + \bar{m}) . \qquad (2.8)$$

The Kronecker delta $\delta_{m-\bar{m},0}$ comes from the boundary conditions in $SL(2,\mathbb{R})/U(1)$, while the Dirac delta-function $\delta(m + \bar{m})$ corresponds to the Neumann boundary conditions for the free boson (and is related to the time-translation symmetry preserved by the brane). The only solution of these antagonistic constraints is $m = \bar{m} = 0$. Using again closed/open channel duality it gives the open string partition function as follows:

$$Z = V_1 \sum_{r \in \mathbb{Z}} ch_{\mathbb{I}}\begin{bmatrix} 0 \\ 0 \end{bmatrix}(r;\tau) \int \frac{\mathrm{d}E}{2\pi} \frac{q^{\frac{-E^2}{k}}}{\eta} \left(\frac{\vartheta\begin{bmatrix} 0 \\ 0 \end{bmatrix}(\tau)}{\eta(\tau)} \right)^{1/2} , \qquad (2.9)$$

where $V_1 \equiv 2\pi\delta(0)$ is the "volume" of the time direction. This open string spectrum looks similar to what we would obtain in a background containing $SL(2,\mathbb{R})/U(1) \times \mathbb{R}^{0,1}$, for instance a non-critical superstring (the divergence coming from the integration over

[2]Details about this standard BCFT computation can be found e.g. in the lecture notes [49], section 3.

[3]On the single cover of $SL(2,\mathbb{R})$ the open string spectrum contains also spectral flowed representations associated with open strings winding around the periodic time direction. This observation allows to check that on the universal cover of $SL(2,\mathbb{R})$, the boundary state corresponds to only one copy of the D-instanton and not an array of them along the Euclidean time direction.

[4]The fermionic characters are written using the theta-function $\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(\tau) = \sum_{n \in \mathbb{Z}} q^{\frac{1}{2}(n+a/2)^2} e^{i\pi(n+a/2)b}$ where $a, b \in \mathbb{Z}_2$ label the different spin structures. $a = 0$ (resp. $a = 1$) corresponds to the Neveu-Schwarz (resp. Ramond) sector. The $b = 1$ sectors have a $(-)^F$ insertion in the trace.

[5]In the following we will not write explicitly the $(z, \bar{z})$ dependence of the one-point functions any more.

space-time energy E has to be regularized as in this flat space-time). We will see below that for the rolling tachyon BCFT describing its decay the $SL(2,\mathbb{R})/U(1)$ and time-like $U(1)$ open string sectors are not decoupled anymore, since closed string primaries with $m \neq 0$ contribute to the annulus amplitude.

2.2 The non-BPS D-particle in $AdS_3 \times S^3 \times T^4$

We consider now the full type IIB superstring theory on $AdS_3 \times S^3 \times T^4$ with NS-NS fluxes. It can be obtained as the near-horizon geometry for a collection of k coincident NS5-branes and Q_1 fundamental strings smeared on the four-torus [51]. The level of the $SL(2,\mathbb{R})$ algebra is k (corresponding to anti-de Sitter space-time of radius $\sqrt{\alpha' k}$), while the six-dimensional string coupling constant is fixed in the near-horizon limit to

$$ g_6 = \frac{g_s}{\sqrt{v_4}} = \sqrt{\frac{k}{Q_1}}\,, \tag{2.10} $$

where v_4 is the T^4 volume in strings units. We will take this solution as a concrete example to embed the non-BPS D-particle, and later the associated rolling tachyon, in a superstring background.

Besides the AdS_3 factor, the three-sphere with NS-NS flux is described as an $SU(2)$ super-WZW model at level k. An important point is that, since this curved background is made only of WZW models, the worldsheet fermions are free even though the background is non-trivial. The space-time supercharges are then constructed exactly as in flat space using the associated spin fields [52], with however the extra condition $\gamma^{012345}\zeta_{L,R} = \zeta_{L,R}$ on the two ten-dimensional spinors, giving overall 16 supercharges [53].

We wish now to consider D-branes in this superstring theory, more specifically to embed the AdS_3 D0-brane discussed above. For the $SU(2)$ part we choose the "elementary" boundary state associated to the identity representation, i.e. the S^2 brane of minimal volume that is viewed as a point on S^3 in the semi-classical limit. Other choices, corresponding to "large" two spheres, can be viewed as bound states of these elementary branes [54]. In order to simplify the notation we take an orthogonal torus. We denote by R_i, $i = 1, \ldots, 4$ the radii of the circles.

There are two reasons for the D0-brane not to be BPS. First, a D0-brane in type IIB has the wrong dimensionality to be supersymmetric, as it doesn't couple to the R-R sector. Second, even in type IIA, this brane would be non-supersymmetric because it breaks the $SL(2,\mathbb{R})$ symmetry which prevents from decoupling the worldsheet fermions and writing the space-time supercharges with spin fields, as was done in [53] for the closed string sector. It is also not possible to use a Gepner-like construction [55] because the R-charges of the $\mathcal{N} = 2$ worldsheet superconformal algebra associated with the $SL(2,\mathbb{R})/U(1)$ coset are not integral. One can check that this D0-brane of type IIA, although non-BPS, is stable (i.e. the open string sector does not contain tachyons).

Let's come back to the non-BPS D0-particle in type IIB superstrings on $AdS_3 \times S^3 \times$

T^4. Its open string partition function reads :

$$Z(\tau) = \sum_{a \in \mathbb{Z}_2} (-)^a \sum_{\{w^i\} \in \mathbb{Z}^4} \frac{q^{\frac{(R_i w^i)^2}{\alpha'}}}{\eta^3(\tau)} \sum_{r \in \mathbb{Z}} ch_{\mathbb{I}} \begin{bmatrix} a \\ 0 \end{bmatrix} (r; \tau) \int \frac{\mathrm{d}E}{2\pi} q^{-\frac{E^2}{k}} \chi^0(\tau) \frac{\vartheta \begin{bmatrix} a \\ 0 \end{bmatrix}^3 (\tau)}{\eta^3(\tau)} . \quad (2.11)$$

where $\chi^0(\tau)$ is the $\widehat{\mathfrak{su}(2)}$ character of the trivial representation. The NS and R sectors correspond to $a = 0$ and $a = 1$ respectively. Let us now study open string tachyonic modes on this D-particle, since we will be ultimately interested in finding the time-dependent solutions associated with rolling down these negative directions of the potential in the open string field theory. By taking the identity in the trivial representation for the $\mathrm{SL}(2, \mathbb{R})/\mathrm{U}(1)$ factor ($r = 0$) as well as for SU(2) we find a tachyon similar to the one that appears on a non-BPS D-brane in flat space-time, with mass squared $m^2_{\text{TACH}} = -1/2\alpha'$ (using flat space-time normalization). If we choose the D-particle to be point-like on the three-sphere, there are no other tachyonic modes provided the compactification torus is large enough. For branes wrapping a non-vanishing S^2, associated with an SU(2) representation of spin $\hat{j}$, there are extra negative modes of mass squared $m^2_{\text{TACH}} = -1/2\alpha' + \tilde{j}(\tilde{j}+1)/\alpha' k$ if $\tilde{j} \in \{1, 2, \dots, 2\hat{j}\}$ is small enough. They correspond to inhomogeneous decay on their two-sphere worldvolume. However the rolling tachyon BCFT is more complicated since the boundary deformation involves a non-trivial boundary operator in the SU(2) sector. In the following we will concentrate on point-like D-particles with $\hat{j} = 0$.

3. The rolling tachyon boundary state in AdS$_3$

Having constructed the non-BPS D-brane of interest in the AdS$_3 \times$ S$^3 \times$ T^4 type IIB superstring background, and having found its tachyonic modes, we will now study the boundary state corresponding to the decay of this D-particle towards the minimum of the tachyon potential, the closed string vacuum. We will first review some aspects of the rolling tachyon boundary conformal field theory in flat space-time, then apply these techniques to the anti-de Sitter case.

3.1 Rolling tachyon in flat space-time: short review

The rolling tachyon is an exact solution of the boundary worldsheet CFT describing the decay of an unstable D-brane [18]. For simplicity we discuss first the bosonic string. It comes in two versions corresponding to the boundary marginal deformations:

$$\delta S_{\text{FULL}} = \lambda \oint_{\partial \Sigma} \mathrm{d}\ell \, \cosh\left(X^0(\ell)/\sqrt{\alpha'}\right) \quad (3.1\text{a})$$

$$\delta S_{\text{HALF}} = \frac{\tilde{\lambda}}{2} \oint_{\partial \Sigma} \mathrm{d}\ell \, e^{X^0(\ell)/\sqrt{\alpha'}} \quad (3.1\text{b})$$

The first one describes a time-dependent process in which incoming radiation conspires – involving a considerable fine-tuning – to create a non-BPS D-brane at $x^0 = 0$ that will eventually decay back to the closed string vacuum. The second one corresponds to the decay of a non-BPS D-brane prepared at past infinity $x^0 \to -\infty$. The parameter $\tilde{\lambda}$ is not

important in this case since it can be absorbed by a time translation. In both cases the decay is homogeneous along the D-brane longitudinal directions.

As always in string theory, in order to study this worldsheet CFT corresponding to a time-dependent process in spacetime we need to use an analytic continuation that is not uniquely defined. It turns out in this case that the rolling tachyon BCFT can be obtained from two rather different types of Euclidean models. The first one, which is more naturally related to the "full S-brane" solution (3.1a), is the boundary sine-Gordon theory [56–59]. It consists in a $c = 1$ free theory of a space-like boson, with the boundary deformation $\lambda \oint_{\partial\Sigma} d\ell \, \cos X(\ell)/\sqrt{\alpha'}$. The construction of the boundary state uses as a basis of Ishibashi states the higher order Virasoro primaries that occur for dimensions $\Delta \in (\mathbb{Z})^2/4$ and the underlying $\widehat{\mathfrak{su}}(2)_1$ symmetry of the model. The Lorentzian theory does not have such primaries (because all the Virasoro representations except the identity are non-degenerate) and such a symmetry, therefore the Wick rotation is quite intricate. Some components of the boundary state obtained this way grow exponentially with time [60] but do not correspond to on-shell physical closed string states. Even though they have been interpreted as conserved charges in the of two-dimensional strings context [61] their interpretation in critical strings and their physical relevance is unclear.

The other route to the rolling tachyon is a particular limit of Liouville theory [19] involving two analytic continuations. Liouville theory has a central charge $c = 1 + 6(b + b^{-1})^2 > 25$ and to get a $c = 1$ theory one should consider the analytic continuation $b \to i$. In this limit the Liouville potential ceases to be a wall and become periodic.[6] For the boundary CFT that we consider, namely Liouville theory with boundary conditions corresponding to an extended D-brane (called FZZT brane [64, 65]) we take the limit of vanishing Liouville potential, while keeping fixed the boundary cosmological constant $\tilde{\lambda}$. The other analytic continuation corresponds to Wick-rotating the Liouville field $\phi \to iX^0$ to get a time-like boson; it gives then the "half-S-brane" solution (3.1b). However the BCFT data, excepting the one-point function, is not analytic in the Liouville momentum for $b \in i\mathbb{R}$ [62, 66].

The one-point function for the FZZT-brane in time-like Liouville theory with $c = 1$ and vanishing bulk cosmological constant reads [19]:

$$\langle e^{ik_0 X^0} \rangle = \frac{1}{\sqrt{2\pi}} \left(\pi\tilde{\lambda} \right)^{-i\sqrt{\alpha'}k_0} \frac{\pi}{\sinh \pi\sqrt{\alpha'}k_0} \,. \tag{3.2}$$

This is what we will consider in the following as defining the coupling between the rolling tachyon (3.1b) and closed strings.[7]

The extension of the above results to the superstring case is rather straightforward, applying the same manipulations $\mathcal{N} = 1$ super-Liouville theory with a boundary. The

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[6]As shown in [62] this unitary CFT is identical to the $c \to 1$ limit of Virasoro minimal models studied in [63].

[7]There is a complication arising for $k_0 = 0$, which is a simple pole of the one-point function, related to the fact that the identity representation is degenerate. It splits into infinitely many Virasoro ireps labeled by $J \in \mathbb{Z}_{\geqslant 0}$, whose highest weight state $|J, 0, 0\rangle$, of dimension J^2, is a polynomial in X^0 derivatives $\mathcal{O}(\partial X^0, \partial^2 X^0, \bar{\partial} X^0, \bar{\partial}^2 X^0, \ldots)$. A more correct definition of the boundary state would be $|B\rangle = \sqrt{\pi/2} \, \mathcal{P} \int dk_0 \, (\pi\tilde{\lambda})^{-i\sqrt{\alpha'}k_0} \sinh^{-1}(\pi\sqrt{\alpha'}k_0)|k_0\rangle\rangle + \sum_{J=0}^{\infty} |J, 0, 0\rangle\rangle$.

boundary deformation associated with the rolling tachyon now reads [67]:

$$\delta S = \frac{\tilde{\lambda}}{2} \oint_{\partial \Sigma} d\ell \, \psi^0 e^{X^0/\sqrt{2\alpha'}} \sigma^1 \qquad (3.3)$$

where σ^1 is a Chan-Patton factor coming from a fermionic zero mode on the worldsheet boundary. It is studied by analytic continuation of the super-FZZT D-brane [68, 69]. Then the one-point function for an NS-NS primary in the presence of this boundary deformation is similar to the bosonic strings result (3.2): [8]

$$\langle e^{ik_0 X^0} \rangle = \frac{1}{\sqrt{2\pi}} \left(\frac{\pi \tilde{\lambda}}{2} \right)^{-i\sqrt{2\alpha'}k_0} \frac{\pi}{\sinh \pi \sqrt{\frac{\alpha'}{2}} k_0} \, . \qquad (3.4)$$

Operators in the R-R sector of the closed superstring theory get non-zero one-point functions once the rolling tachyon perturbation is turned on. More explicitly, we consider the spin fields σ^ϵ, with $\epsilon = \pm 1$, creating the two Ramond ground states for ψ^0, the worldsheet fermionic super-partner of X^0. The one-point function in the R-R sector depends on the two possible gluing conditions for the $\mathcal{N} = 1$ supercurrent on the real axis:

$$G(z) = \xi \tilde{G}(\bar{z})|_{z=\bar{z}} \quad , \qquad \xi = \pm 1 \, . \qquad (3.5)$$

For both signs, the one-point function for the super-FZZT-brane in time-like super-Liouville theory with $c = 3/2$ and vanishing bulk cosmological constant reads:

$$\langle e^{ik_0 X^0(z,\bar{z})} \sigma^\epsilon(z) \tilde{\sigma}^{\bar{\epsilon}}(\bar{z}) \rangle = \frac{\epsilon^{\frac{\xi+1}{2}}}{\sqrt{2\pi}} \left(\frac{\pi \tilde{\lambda}}{2} \right)^{-i\sqrt{2\alpha'}k_0} \frac{\pi}{\cosh \pi \sqrt{\frac{\alpha'}{2}} k_0} \delta_{\epsilon,\bar{\epsilon}} \, . \qquad (3.6)$$

Recall that a non-BPS D-brane with constant open string tachyon profile does not couple to the R-R fields. In the effective action the couplings between the tachyon field and the R-R forms are derivative.

3.2 D0-brane decay in AdS$_3$

We have now prepared the ground to construct the boundary state of the rolling tachyon in AdS$_3$. We follow the approach based on time-like Liouville theory that we reviewed above.

Let us first recall that we showed in the previous section that the unstable D0-brane in type IIB superstring theory on AdS$_3 \times$ S$_3 \times$ T^4 contains on its worldvolume an open string tachyon built with the *identity* of the SL$(2,\mathbb{R})$/U(1) super-coset. Therefore one can turn on the same boundary deformation (3.3) that was discussed above in flat-space-time, built with the identity of the SL$(2,\mathbb{R})$/U(1) conformal field theory. According to the general analysis of boundary deformations performed in [58], this deformation of the AdS$_3$ superstring is exactly marginal, therefore it gives an exact rolling tachyon solution of the open string field theory. Moreover we shall see below that the one-point function giving the coupling

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[8]See [70] for another derivation of this result using free field correlators.

between closed string modes and the rolling tachyon can be computed exactly. To simplify the equations we will consider till the end of this section only the $SL(2,\mathbb{R})$ part of the one-point functions. In the full $AdS_3 \times S_3 \times T^4$ background the one-point function will be the product of the $SL(2,\mathbb{R})$ piece with the $SU(2) \times U(1)^4$ piece which is standard, together with the appropriate GSO projection.

As reviewed in more detail in appendix A, the primary states in the Minkowskian $SL(2,\mathbb{R})$ theory – characterized by their $SL(2,\mathbb{R})$ spin j, the eigenvalues $(m, \bar{m})$ of the elliptic sub-algebras J_0^3 and $\bar{J}_0^3$ and the spectral flow sector w – are defined as primary states in the "T-dual" theory $SL(2,\mathbb{R})/U(1) \times \mathbb{R}^{0,1}$. For NS-NS primaries we have the decomposition:

$$V_{m\bar{m}w}^{j,\mathfrak{sl}_2} = V_{m - \frac{kw}{2}; \; -\bar{m} + \frac{kw}{2}}^{j,\mathfrak{sl}_2/\mathfrak{u}_1} \; e^{i\sqrt{\frac{2}{k}}\left[mX^0(z) + \bar{m}\tilde{X}^0(\bar{z})\right]} \tag{3.7}$$

where $X^0(z, \bar{z})$ is a canonically normalized free time-like boson (with $\alpha' = 2$). To deal with the R-R sector it is convenient to bosonize the fermionic superpartners of the currents $J^\pm$ and $\bar{J}^\pm$ as $2i\partial H_1 = \psi^+\psi^-$ and $2i\bar{\partial}\tilde{H}_1 = \tilde{\psi}^+\tilde{\psi}^-$. Then we have the following decomposition:

$$V_{m\bar{m}w}^{j,\mathfrak{sl}_2} e^{\frac{i}{2}(sH_1 + \bar{s}\tilde{H}_1)}\sigma^{\epsilon,\bar{\epsilon}} = V_{m - \frac{kw}{2}; \; -\bar{m} + \frac{kw}{2}}^{j,\mathfrak{sl}_2/\mathfrak{u}_1 \, (s,\bar{s})} \; e^{i\sqrt{\frac{2}{k}}\left[mX^0(z) + \bar{m}\tilde{X}^0(\bar{z})\right]}\sigma^{\epsilon,\bar{\epsilon}}. \tag{3.8}$$

with $s, \bar{s} = \pm 1$. The one-point function for a D-particle, in the presence of the boundary deformation (3.3), can be obtained as follows. First, due to the decomposition (3.7) of $SL(2,\mathbb{R})$ vertex operators into an $SL(2,\mathbb{R})/U(1)$ piece and a free boson piece, the contribution to the rolling tachyon one-point function from the $SL(2,\mathbb{R})/U(1)$ vertex operator will be the same as its contribution to the D(-1)-brane, see eq. (2.6), up to the overall normalization factor. Indeed the boundary deformation (3.3) corresponding to the rolling tachyon is built with the *identity* operator of $SL(2,\mathbb{R})/U(1)$ CFT and therefore cannot change the $SL(2,\mathbb{R})/U(1)$ contribution to the coefficients of the boundary state.

Second, the contribution from the time-like boson is identical, for a given value of the spacetime energy, to the one-point function for the rolling tachyon in flat space-time (3.2). Indeed this is a free boson, in the sense that, expanding an $\widehat{\mathfrak{sl}}(2,\mathbb{R})$ highest weight module in terms of the horizontal Cartan subalgebra J_0^3, for a given value of m the sub-module splits into an $\widehat{\mathfrak{sl}}(2,\mathbb{R})/\widehat{\mathfrak{u}}_1$ highest weight module and a free $\widehat{\mathfrak{u}}_1$ one. Moreover the $SL(2,\mathbb{R})/U(1)$ contribution to the one-point function is analytic in m so one can choose imaginary values corresponding to "resonances" for which the Liouville couplings are given by free field computations.

Assembling everything together we get the one-point function in the presence of the decaying D-particle in AdS_3. It leaves a freedom for the overall normalization of the one-point function, that we fixed by an analogue of the Cardy condition.[9] Explicitly, we get first the one-point function for an NS-NS primary as follows:

$$\langle V_{E/2 \; E/2 \; w}^{j,\mathfrak{sl}_2} \rangle = \frac{\pi}{(2k)^{3/4}} \, \nu_k^{1/2-j} \, \frac{\Gamma\left(j + \frac{kw-E}{2}\right)\Gamma\left(j - \frac{kw-E}{2}\right)}{\Gamma(2j-1)\Gamma(1 + \frac{2j-1}{k})} \, \frac{\left(\frac{\pi\tilde{\lambda}}{2}\right)^{-i\sqrt{\frac{2}{k}}E}}{\sinh\frac{\pi E}{\sqrt{2k}}}. \tag{3.9}$$

---

[9]One requires that the annulus amplitude in the open string channel has a normalization compatible with the regularized density of open string states [65].

This is one of the main results of the paper, that we will use in the following to study the physics of the decay. Compared to the static D0-brane one-point function, eq. (2.8), there is no constraint $\delta(E)$ because the brane is time-dependent. Single closed string states with angular momentum (i.e. $m - \bar{m} \neq 0$) don't couple to the D0-brane, because the latter carries no angular momentum in its rest frame.

One can find the profile of the closed string wave-function for the brane by inverse Fourier transform in the sector $w = 0$, using a basis of (delta-function)-normalizable functions on the $SL(2, \mathbb{R})$ group manifold, i.e. matrix elements in discrete and continuous representations. The one-point function (3.9) is the product of the one-point function for a D(-1) instanton and for the rolling tachyon in flat space-time; therefore the profile is obtained by the convoluted product of these two. In the semi-classical limit, i.e. neglecting the factor $\Gamma(1 + 2j^{-1}/k)$ and removing the upper bound (2.3) on j, the Fourier transform of the $D(-1)$ coupling is proportional to $\delta(\rho)\delta(t)$. We find that the spacetime profile of the rolling tachyon in the semi-classical limit is

$$\Psi(\rho, t, \phi) \propto \frac{\delta(\rho)}{1 + (\frac{\pi\tilde{\lambda}}{2})^2 e^{\sqrt{2k}t}}. \tag{3.10}$$

It describes the decay of an unstable D-particle sitting at the center of AdS$_3$, prepared at past infinity $t \to -\infty$ in global time. We will use later the same one-point function to compute the emission of closed strings coupling to this time-dependent brane.

Let us now move to the R-R sector. With the conventions set above, see eq. (3.8), the one-point function for a R-R primary of $SL(2, \mathbb{R})$ in the presence of the rolling tachyon reads:

$$\langle V_{E/2\,E/2\,w}^{j,\mathfrak{sl}_2} e^{\frac{i}{2}(sH_1 + \bar{s}\tilde{H}_1)} \sigma^{\epsilon,\bar{\epsilon}} \rangle = \epsilon^{\frac{\xi+1}{2}} e^{-\frac{i\pi s\hat{s}}{2}} \frac{\pi}{(2k)^{3/4}} \nu_k^{1/2-j} \times$$

$$\times \frac{\Gamma\left(j + \frac{kw-E+s}{2}\right)\Gamma\left(j - \frac{kw-E-\bar{s}}{2}\right)}{\Gamma(2j-1)\Gamma(1 + \frac{2j-1}{k})} \frac{\left(\frac{\pi\tilde{\lambda}}{2}\right)^{-i\sqrt{\frac{2}{k}}E}}{\cosh\frac{\pi E}{\sqrt{2k}}} \delta_{s,-\bar{s}}\delta_{\epsilon,\bar{\epsilon}} \tag{3.11}$$

with a parameter $\hat{s} \in \mathbb{Z}_4$ such that $\hat{s}$ is even for $\xi = 1$ and odd otherwise. In the full supersymmetric type IIB background the boundary state is a linear combination of the $\xi = 1$ and $\xi = -1$ boundary states in the NS-NS and R-R sectors in the usual way compatible with the closed string GSO projection [71].

We recall that this one-point function contains both couplings to the discrete representations (the residues of the simple poles on the real axis in the j-plane) and couplings to the continuous representations with $j = \frac{1}{2} + iP$. Therefore one of the most prominent features of the the decay of this D-brane is that it produces long strings that "escape" to space-like infinity. In the following section we will make this more precise and compute the emission of closed strings as the imaginary part of the annulus amplitude.

Conserved energy. The conserved energy of the rolling tachyon can be found from the "conservation law" for the boundary state:

$$(Q_B + \bar{Q}_B)|B\rangle = 0, \tag{3.12}$$

in order to satisfy the closed string field theory equations of motion; Q_B and $\bar{Q}_B$ are the left and right contribution to the BRST charge. We refer the reader to the review [15] for more details and references. The expansion of the boundary state for the rolling tachyon in an $AdS_3 \times \mathcal{M}$ bosonic string background at level k contains the terms

$$
|B\rangle \propto \int \, \mathrm{d}j \; \nu_{k-2}^{1/2-j} \sum_w \int \mathrm{d}E \; \frac{\Gamma\left(j + \frac{kw-E}{2}\right)\Gamma\left(j - \frac{kw-E}{2}\right)}{\Gamma(2j-1)\Gamma(1 + \frac{2j-1}{k-2})}
$$
$$
\left(\frac{\pi(\pi\lambda)^{-\frac{iE}{\sqrt{k}}}}{\sinh\frac{\pi E}{\sqrt{k}}} + \frac{2}{k}\left[4\pi\delta(E/\sqrt{k}) - \frac{\pi(\pi\lambda)^{-\frac{iE}{\sqrt{k}}}}{\sinh\frac{\pi E}{\sqrt{k}}}\right] J_{-1}^3 \bar{J}_{-1}^3 + \cdots\right)|j\, E/2\, E/2\, w\rangle\!\rangle
$$
$$
\otimes |B\rangle_\mathcal{M} \otimes (1 - \tilde{b}_{-1}c_{-1} - b_{-1}\tilde{c}_{-1} + \cdots)(c_0 + \bar{c}_0)c_1\bar{c}_1|0\rangle_{gh}\,, \quad (3.13)
$$

with contributions both from the continuous and discrete representations of $\mathfrak{sl}(2,\mathbb{R})$. Isolating the component proportional to $(\tilde{c}_{-1}J_{-1}^3 + c_{-1}\bar{J}_{-1}^3)$ in (3.12) we find the conserved energy associated with the rolling tachyon in AdS$_3$, expressed in momentum space as follows:

$$
T_{00}(j,w) \propto \frac{1}{g_s} \, \nu_{k-2}^{1/2-j} \frac{\Gamma\left(j + \frac{kw}{2}\right)\Gamma\left(j - \frac{kw}{2}\right)}{\Gamma(2j-1)\Gamma(1 + \frac{2j-1}{k-2})}\,. \tag{3.14}
$$

Because the brane is point-like the stress-energy tensor has no components longitudinal to the brane worldvolume. We did not calculate the components transverse to the brane, that vanish in flat space-time.

One can find the tachyon energy profile in space-time by inverse Fourier transform in the sector $w = 0$. It is identical (up to a normalization constant) to the wave-function corresponding to the D-particle in AdS$_3$. In the semi-classical limit, we find $T_{00}(\rho, \phi) \propto g_s^{-1}\delta(\rho)$, so the tachyon energy density is sharply localized and constant in global time. For finite k the energy profile is smeared along the radial direction ρ on a scale of order $\sqrt{\alpha'/k}$.

3.3 Rolling tachyon in AdS$_3$ from an orbifold construction

The coefficients of the boundary state for the rolling tachyon in AdS$_3$ – or, in other words, the one-point function – can be obtained alternatively using an orbifold construction. This approach will prove later to be useful to analyze the open string sector of the theory. We start with a slightly unusual T-dual representation of string theory on AdS$_3$ as the orbifold

$$
\mathrm{AdS}_3 \sim \frac{\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)_\infty^{\mathrm{v}} \times \mathbb{R}^{0,1}}{\mathbb{Z}}\,. \tag{3.15}
$$

The first factor in this decomposition is the vector coset $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ of the *universal cover* of $\mathrm{SL}(2,\mathbb{R})$, i.e. the universal cover of the "trumpet" geometry.[10] The latter has a metric $\mathrm{d}s^2 = \mathrm{d}\rho^2 + \coth^2(\rho/\sqrt{2k})\mathrm{d}x^2$ with x non-compact. In the coordinates $\mathrm{d}s^2 = 2k\,\mathrm{d}z\mathrm{d}\bar{z}/(z\bar{z}-1)$, one sees that it is conformal to the infinite cover of the exterior

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[10]It is T-dual to a $\mathbb{Z}$ orbifold of the "cigar" [72], i.e. of the axial coset. Another representation of this CFT with a singular target space is the $\mathcal{N} = 2$ Liouville theory with a momentum condensate, at infinite radius [73].

of the unit disc. The action of the $\mathbb{Z}$ translation orbifold on $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)^{\mathrm{V}}_\infty \times \mathbb{R}^{0,1}$, giving the AdS$_3$ CFT is

$$\mathfrak{T} : \quad (x,t) \longrightarrow (x + 2\pi\sqrt{2/k}, t + 2\pi\sqrt{2/k}). \tag{3.16}$$

Since it has no fixed points one can get the D-branes on AdS$_3$ from the branes of the CFT $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)^{\mathrm{V}}_\infty \times \mathbb{R}^{0,1}$ by summing over the images under the orbifold action.

We start on the covering space of the orbifold with the tensor product of a D0-brane in the trumpet and a Neumann brane along the time direction with the rolling tachyon boundary deformation (3.3). The former is an A-type brane of the vector coset $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ which is in some sense localized on the boundary of the disc, in the strong coupling region, in close analogy with the minimal lenght D1-branes of $\mathrm{SU}(2)/\mathrm{U}(1)$ stretched between adjacent "special points" on the boundary of the disc [50]. It carries a label r_0 giving its quantized position, with $r_0 \in \mathbb{Z}$ for the universal cover of the manifold,[11] with a coupling to the x-momentum of the coset (that we parameterize as $p_x = \mu\sqrt{2/k}$) in the one-point function of the form $\exp(4i\pi\mu r_0/k)$.

We now construct the boundary state in the orbifold theory by summing over the images. The action of the translation generator $\mathfrak{T}$ on the labels of the brane is:

$$\mathfrak{T} : \quad r_0 \longrightarrow r_0 + 1 \quad , \qquad \tilde{\lambda} \longrightarrow \tilde{\lambda} e^{\pi\sqrt{\frac{2}{k}}} . \tag{3.17}$$

The transformation of the boundary cosmological constant $\tilde{\lambda}$ follows from the action of the translation (3.16) on the boundary action (3.3). We get the following boundary state (for the NS-NS sector):

$$|B\rangle\!\rangle = \frac{1}{k\pi} \sum_{h\in\mathbb{Z}} \int \mathrm{d}j\, \mathrm{d}\mu\, \nu_k^{1/2-j}\, \frac{\Gamma(j+\mu)\Gamma(j-\mu)}{\Gamma(2j-1)\Gamma(1+\frac{2j-1}{k})}\, e^{\frac{4i\pi\mu h}{k}} \times$$

$$\times\ \frac{1}{\sqrt{2k\pi}} \int \frac{\mathrm{d}E}{2\pi} \left(\frac{\pi\tilde{\lambda}}{2} e^{\pi\sqrt{\frac{2}{k}}h} \right)^{-i\sqrt{\frac{2}{k}}E} \frac{\pi}{\sinh\frac{\pi E}{\sqrt{2k}}}\, |j,\mu,\mu\rangle\!\rangle \otimes |E\rangle\!\rangle$$

$$\tag{3.18}$$

where $|j,\mu,\mu\rangle\!\rangle$ and $|E\rangle\!\rangle$ are respectively the Ishibashi states for the $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ vector super-coset with type A boundary conditions and for the time-like direction (with the same normalization $\Delta = -E^2/4k$ as in AdS$_3$). Now performing the sum over h (i.e. over the images under the orbifold action) one finds a boundary state whose coefficients are identical, up to an overall normalization, to the one-point function (3.9) for the rolling tachyon in AdS$_3$.

4. Closed and open string emission by the rolling tachyon

The physics of the rolling tachyon is, from the closed strings point of view, the decay of an unstable, very massive particle, producing radiation of closed strings composed of all the

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[11]For the single cover of the trumpet $r_0 \in \mathbb{Z}_k$. It reflects the momentum non-conservation in the trumpet, T-dual to the winding non-conservation in the cigar.

string modes coupling to the brane. As recalled in the introduction the average number of closed strings emitted by the D0-brane decay in flat space-time, computed in the tree-level approximation, is divergent [23]. This is the detailed outcome of the competition between the exponential suppression of the emission probability at high energy, and the exponential growth of the density of string modes. Physically, because the initial energy is finite and equal to the mass of the D-particle of order $1/\sqrt{\alpha'}g_s$, one must impose a cutoff at the corresponding energy, which is exactly the value for which non-perturbative effects kick in. Then one concludes that the preferred decay channels are non-relativistic very massive excited closed strings. The decay of higher dimensional branes produces a "tachyon dust" of pressure-less fluid.

In AdS$_3$ we may expect a different picture, as the density of states at high energy behaves like a field theory (because string theory in AdS$_3$ is dual to a two-dimensional conformal field theory). We show below that "long strings" play a prominent role in the closed strings description of the decay. We identify non-perturbative effects that do regularize ultraviolet divergences and give finite physical quantities resulting from the decay. This however does not challenge Sen's conjecture [26] since all the D0-brane energy is converted into closed strings radiation, as will be explained in this section.

4.1 Annulus amplitude for the decaying D-particle

The mean number of emitted closed strings can be obtained from the imaginary part of the annulus diagram, using an optical-like theorem and open/closed channel duality. We follow closely the computation of [34], and the recent analysis of [74] that clarified the issue of the analytic continuation. It gives also an analogue of the Cardy consistency condition [75, 65] for the boundary state.

The annulus amplitude in the closed string channel, computed from the one-point function, gets contributions both from the continuous and from the discrete representations. For simplicity of the notations in the following manipulations we keep track only of the continuous representations in the NS sector; one can check that the contributions of the discrete representations and the R sector of the full superstring background are consistent with the open string annulus amplitude (4.10) that we get at the end of the computation. From the expression of the one-point function, see eq. (3.9), we obtain the closed string channel integrand of the annulus amplitude, in the NS sector of continuous representations as (with $\tilde{q} = \exp 2i\pi\tilde{\tau}$):

$$Z(\tilde{\tau}) = \frac{i\pi}{2}\sqrt{\frac{2}{k}}\sum_{w\in\mathbb{Z}}\int_{-\infty}^{\infty}\frac{\mathrm{d}E}{2\pi}\int_0^{\infty}\mathrm{d}P\,\frac{\sinh 2\pi P \sinh\frac{2\pi P}{k}}{[\cosh 2\pi P + \cos\pi(E-kw)]\sinh^2\frac{\pi E}{\sqrt{2k}}}\times$$

$$\times\,ch_c\begin{bmatrix}0\\0\end{bmatrix}\left(P,\frac{E-kw}{2};\tilde{\tau}\right)\frac{\tilde{q}^{-\frac{E^2}{4k}}}{\eta(\tilde{\tau})}\left(\frac{\vartheta\begin{bmatrix}0\\0\end{bmatrix}(\tilde{\tau})}{\eta(\tilde{\tau})}\right)^{1/2}.\quad(4.1)$$

To obtain a well-defined expression we should consider, as explained in detail in [74], the Lorentzian cylinder with the appropriate ε-prescriptions, i.e. take the modular parameter $\tilde{\tau} = 1/t + i\varepsilon$, $t \in \mathbb{R}_+$ and regularize the integration over the energy using the prescription $E \to E_{-\hat{\varepsilon}} \equiv (1 - i\hat{\varepsilon})E$.

To be able to modular transform to the open string channel we need to follow some algebraic manipulations. We will extract at the end the imaginary part of the annulus amplitude that can be interpreted as the closed string emission. The first step of the computation is to disentangle the $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ part and the $\mathrm{U}(1)$ part of the closed strings channel amplitude. We rewrite (4.1) as follows

$$Z(\tilde{\tau}) = \frac{i}{4} \left(\frac{2}{k} \right)^{3/2} \sum_w \int \mathrm{d}\mu \, \mathrm{d}E \int_0^\infty \mathrm{d}P \frac{\sinh 2\pi P \sinh \frac{2\pi P}{k}}{[\cosh 2\pi P + \cos 2\pi\mu] \sinh^2 \frac{\pi E}{\sqrt{2k}}} \times$$

$$\times ch_c \begin{bmatrix} 0 \\ 0 \end{bmatrix} (P,\mu;\tilde{\tau}) \frac{\tilde{q}^{-\frac{E_{-\hat{\varepsilon}}^2}{4k}}}{\eta(\tilde{\tau})} \delta \left(\frac{2\mu}{k} + w - \frac{E}{k} \right) \left(\frac{\vartheta \begin{bmatrix} 0 \\ 0 \end{bmatrix}(\tilde{\tau})}{\eta(\tilde{\tau})} \right)^{1/2} \tag{4.2a}$$

$$= \frac{i}{4} \left(\frac{2}{k} \right)^{3/2} \int \mathrm{d}\mu \sum_{r\in\mathbb{Z}} \int_0^\infty \mathrm{d}P \frac{\sinh 2\pi P \sinh \frac{2\pi P}{k}}{\cosh 2\pi P + \cos 2\pi\mu} e^{-\frac{4i\pi r\mu}{k}} ch_c \begin{bmatrix} 0 \\ 0 \end{bmatrix} (P,\mu;\tilde{\tau}) \times$$

$$\times \int \frac{\mathrm{d}E}{\sinh^2 \left(\pi E/\sqrt{2k} \right)} e^{+\frac{2i\pi rE}{k}} \frac{\tilde{q}^{-\frac{E_{-\hat{\varepsilon}}^2}{4k}}}{\eta(\tilde{\tau})} \left(\frac{\vartheta \begin{bmatrix} 0 \\ 0 \end{bmatrix}(\tilde{\tau})}{\eta(\tilde{\tau})} \right)^{1/2}. \tag{4.2b}$$

We recognize in the first line of (4.2b) the continuous part of the modular transformation from the identity representation characters of $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ [44, 47, 45] (with $\tau = -1/\tilde{\tau}$):

$$ch_{\mathbb{I}} \begin{bmatrix} 0 \\ 0 \end{bmatrix}(r;\tau) = \frac{4}{k} \left\{ \int_0^\infty \mathrm{d}P \int \mathrm{d}\mu \, e^{-4i\pi\frac{\mu r}{k}} \frac{\sinh 2\pi P \sinh \frac{2\pi P}{k}}{\cosh 2\pi P + \cos 2\pi\mu} ch_c \begin{bmatrix} 0 \\ 0 \end{bmatrix}(P,\mu;\tilde{\tau}) + \text{discrete} \right\} \tag{4.3}$$

where the second part term contains an integral over characters in the discrete representations of $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ with spin in the range $1/2 < j < {}^{k+1}/2$. One can check that this discrete part of the modular transform matches the contribution of the discrete representations to the closed string channel annulus amplitude that we get from residues at the poles of the one-point function (3.9).

Then we have to deal with the $\mathrm{U}(1)$ part of (4.2b), i.e. the integral over space-time energy. In terms of the open string channel modulus $\tau = t - i\varepsilon$ (regularized by the ε prescription), we get the modular transformation (with $q = \exp 2i\pi\tau$):

$$\int \frac{\mathrm{d}E}{\sinh^2 \left(\pi E/\sqrt{2k} \right)} e^{\frac{2i\pi rE}{k}} \frac{\tilde{q}^{-\frac{E_{-\hat{\varepsilon}}^2}{4k}}}{\eta(\tilde{\tau})} = \int \mathrm{d}\upsilon \frac{q^{-\frac{\upsilon_{\hat{\varepsilon}}^2}{2}}}{\eta(\tau)} \int \mathrm{d}E \frac{\cos \frac{2\pi E}{k} \left(\sqrt{\frac{k}{2}} \, \upsilon + r \right)}{\sinh^2 \left(\pi E/\sqrt{2k} \right)}, \tag{4.4}$$

with as before $\upsilon_{\hat{\varepsilon}} \equiv (1+i\hat{\varepsilon})\upsilon$. Finally, adding the contribution from $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ discussed above we get the open string channel amplitude as:

$$Z(\tau) = \frac{i}{8} \sqrt{\frac{2}{k}} \sum_{r\in\mathbb{Z}} ch_{\mathbb{I}}(r;\tau) \int \mathrm{d}\upsilon \frac{q^{-\frac{\upsilon_{\hat{\varepsilon}}^2}{2}}}{\eta(\tau)} \times \left[\int \mathrm{d}E \frac{\cos \frac{2\pi E}{k} \left(\sqrt{\frac{k}{2}} \, \upsilon + r \right)}{\sinh^2 \left(\pi E/\sqrt{2k} \right)} \right]. \tag{4.5}$$

As expected, the density of states given by the bracketed expression is divergent (it has a double pole at $E = 0$); this infrared divergence is due to the infinite "volume" along the time direction as in ordinary Liouville theory. We can subtract this double pole, that

will not contribute to the imaginary part of the amplitude, and then introduce the special function $S_\beta^{(0)}(x)$ [64]:

$$\ln S_\beta^{(0)}(x) = \frac{1}{2} \int_0^\infty \frac{\mathrm{d}y}{y} \left[\frac{\sinh(Q_\beta - 2x)y}{\sinh \beta y \ \sinh \beta^{-1} y} + \frac{2x - Q_\beta}{y} \right] , \qquad (4.6)$$

with $Q_\beta = \beta + \beta^{-1}$. In terms of this function we obtain the open string channel annulus amplitude for the decaying D-particle in AdS$_3$ as follows:

$$Z(\tau) = \sum_{r \in \mathbb{Z}} ch_{\mathbb{I}} \begin{bmatrix} 0 \\ 0 \end{bmatrix} (r; \tau) \int \mathrm{d}\upsilon \, \frac{1}{2\pi} \frac{\mathrm{d}}{\mathrm{d}\upsilon} \ln S_1^{(0)} \left(1 - i\upsilon - i\sqrt{\frac{2}{k}} \, r \right) \frac{q^{-\frac{\upsilon_\varepsilon^2}{2}}}{\eta(\tau)} \left(\frac{\vartheta \begin{bmatrix} 0 \\ 0 \end{bmatrix}(\tau)}{\eta(\tau)} \right)^{1/2} . \quad (4.7)$$

Let us now consider the physical setup of interest, namely type IIB strings on AdS$_3 \times$ S$^3 \times$ T^4 in the presence of the D-particle. Let us remind that we have chosen to take a D0-brane in the S^3 part of the background, i.e. the symmetric brane of SU(2) with $\hat{\jmath} = 0$ (which implies that only the $\tilde{\jmath} = 0$ representation is possible in the open string sector) and Dirichlet boundary conditions along all the cycles of the four-torus. There are also extra contributions from the twisted Neveu-Schwarz (i.e. with a $(-)^F$ insertion) and Ramond sectors to the annulus amplitude in type II superstrings. To write the density of states in the open string channel twisted NS sector (coming from the closed string channel R sector) it is useful to define another special function, which turns out to have no pole to subtract at the origin:

$$\ln S_\beta^{(1)}(x) = \frac{1}{2} \int_0^\infty \frac{\mathrm{d}y}{y} \frac{\sinh(Q_\beta - 2x)y}{\cosh \beta y \ \cosh \beta^{-1} y} . \qquad (4.8)$$

Once adding everything and integrating over the modular parameter τ we get the integrated annulus amplitude for the rolling tachyon in AdS$_3 \times$ S$^3 \times$ T^4:

$$\mathcal{A} = \frac{1}{2} \sum_{a,b \in \mathbb{Z}_2} (-)^b \int \frac{\mathrm{d}t}{2t} \sum_{r \in \mathbb{Z}} ch_{\mathbb{I}} \begin{bmatrix} b \\ a \end{bmatrix} (r; \tau) \times \qquad (4.9)$$

$$\times \int \mathrm{d}\upsilon \frac{1}{2\pi} \frac{\mathrm{d}}{\mathrm{d}\upsilon} \ln S_1^{(a)} \left(1 - i\upsilon - i\sqrt{\frac{2}{k}}(r + b/2) \right) q^{-\frac{\upsilon_\varepsilon^2}{2}} \chi^0(\tau) \sum_{\{w^i\}} \frac{q^{\frac{(R_i w^i)^2}{2}}}{\eta^3(\tau)} \frac{\vartheta \begin{bmatrix} b \\ a \end{bmatrix}^3 (\tau)}{\eta^3(\tau)} .$$

From this result we will derive in the following subsection the production of closed strings as the imaginary part of the annulus amplitude.

As in ordinary Liouville theory the "energy density" $\partial_\upsilon \ln S_1^{(a)}(1 - i\upsilon - ir\sqrt{2/k})$ is related to the boundary two-point function [65]. This provides a consistency check for the boundary state, analogous to the Cardy condition. The annulus amplitude contains first a contribution from the "tachyon sector", i.e. open strings with odd fermion number whose lowest state is the open string tachyon. The associated spectral density is written in terms of

$$\ln S_\beta^{\mathrm{T}}(x) = \frac{1}{2} \left(\ln S_\beta^{(0)}(x) + \ln S_\beta^{(1)}(x) \right) \qquad (4.10)$$

$$= \int_0^\infty \frac{\mathrm{d}y}{y} \left[\frac{\sinh(\frac{Q_\beta}{2} - x)y \ \cosh \frac{Q_\beta y}{2}}{\sinh \beta y \sinh \beta^{-1} y} + \frac{x - \frac{Q_\beta}{2}}{y} \right] = \ln S_\beta^{(0)} \left(\frac{x}{2} \right) S_\beta^{(0)} \left(\frac{x + Q_\beta}{2} \right) .$$

This expression is closely related to the boundary two-point function of super-Liouville theory given in [69]. We will discuss this issue in more detail in subsection 4.4. Similarly for the "vector sector", i.e. open string states with even fermion number, the spectral density is written using

$$\ln S_\beta^{\mathrm{v}}(x) = \frac{1}{2}\left(\ln S_\beta^{(0)}(x) - \ln S_\beta^{(1)}(x)\right) = \ln S_\beta^{(0)}\left(\frac{x+\beta}{2}\right) S_\beta^{(0)}\left(\frac{x+\beta^{-1}}{2}\right). \qquad (4.11)$$

For the Ramond sector a similar decomposition according to the fermion number can be carried out; however it is not useful in order to analyze the annulus amplitude since the contribution from the twisted Ramond sector is identically zero.

4.2 Analysis of the closed string emission

As in [34, 74] we will now extract the imaginary part of this annulus amplitude, that appears once we Wick-rotate the expression (4.10) to Euclidean signature $v \to iv$.[12] It gives the mean number of produced on-shell closed string states as we will check later by interpreting this quantity in the closed string channel.

The imaginary part of the annulus diagram comes from the simple poles of $S_\beta^{(0)}(x)$ in the densities (4.10), (4.11) (and also in the R sector), located at $x = -n\beta - m\beta^{-1}$, for $m, n \in \mathbb{Z}_{\geqslant 0}$, and the simple zeroes for $m, n \in \mathbb{Z}_{<0}$. For $\beta = 1$ both give double poles of the energy density. Compared to the flat space-time computation, with poles only on the imaginary axis, the partition function contains new poles all over the v-plane in the sectors $r \neq 0$, much like in the annulus for the accelerating D-brane in NS5-backgrounds studied in [74]. The residues of the poles situated in the upper-right and lower-left quadrant of the v-plane, including the imaginary axis, will then contribute to the imaginary part of the annulus amplitude. Note finally that these poles don't correspond to on-shell physical open string states; the associated open string wave-functions decrease exponentially with time towards past infinity. They correspond rather to on-shell closed string states as will be shown below using channel duality.

Adding the contributions from the different sectors and using the symmetry (A.8) defined in appendix A, we obtain from (4.10) the imaginary part of the annulus amplitude, written in the open string channel, as the sum over the residues:

$$\operatorname{Im}\mathcal{A} = \frac{1}{2}\int_0^\infty \frac{dt}{2t}\sum_{a,b\in\mathbb{Z}_2}(-)^b\chi^0(it)\,\frac{\vartheta{\left[{b\atop a}\right]}^3(it)}{\eta^3(it)}\sum_{\{w^i\}}\frac{e^{-\pi t(R_i w^i)^2}}{\eta^3(it)}\sum_{r\in\mathbb{Z}}ch_{\mathbb{I}}{\left[{b\atop a}\right]}(r;it) \times$$
$$\times \sum_{n=1}^\infty (-)^{a(n+1)}\,n\,e^{-\pi t\left(n-i\sqrt{\frac{2}{k}}(r+b/2)\right)^2}. \qquad (4.12)$$

The sign $(-)^{a(n+1)}$ in the last line comes from the analysis of the poles in the open string twisted NS sector. Note that the sum over r is convergent because we get a factor of $e^{-2\pi t\{(r+b/2)^2/k+|r|\}}$ from the identity character $ch_{\mathbb{I}}(r;it)$ of $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$. As we show below, this expression gives the distribution of closed string emission, as it is expected

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[12]Together with a Wick rotation of the Schwinger parameter t.

using an optical-like theorem and open/closed string duality of the annulus diagram [34]. To obtain a proper closed strings interpretation of this quantity we need to modular transform back to the closed string channel; the computation is given in appendix B.

4.2.1 Long strings production

We show that the mean number of produced closed strings is equal to the imaginary part of the annulus diagram given by eq. (4.12), looking first at the emission of long closed strings. The initial condition for closed strings at past infinity set by the rolling tachyon worldsheet CFT corresponds to the non-BPS D0-brane boundary state, in the absence of closed string radiation. We may expect that the production of very massive long strings is highly suppressed, because the density of states at high energies is significantly lower than in flat space-time (because the energy scales like $E \sim N/w$ with the oscillator number N, compared to $E \sim \sqrt{N}$ in flat space-time). We turn now to the exact computation that partially confirms these expectations.

After a modular transformation of the imaginary part of the annulus amplitude (4.12) to the closed string channel, one get first a contribution from the continuous $\mathrm{SL}(2,\mathbb{R})$ representations:

$$
\mathrm{Im}\mathcal{A}_c = \frac{1}{4k\prod_i R_i} \sum_{|\phi\rangle\in\mathcal{H}_c} \sum_{2\tilde{j}=1}^{k-2} \sin\left(\frac{\pi(2\tilde{j}+1)}{k}\right) \sum_{\{n_i\}\in\mathbb{Z}} \int \mathrm{d}P \sum_{a,b} \sum_N (-)^{bF} D(N) \times
$$
$$
\times \sum_{w\neq 0} \frac{1}{|w|} \left[\frac{\sinh 2\pi P \sinh\frac{2\pi P}{k}}{\cosh 2\pi P + \cos\pi(kw - E_a)}\right] \frac{(-)^a}{\sinh^2\left(\frac{\pi E_a}{\sqrt{2k}} + \frac{i\pi a}{2}\right)} \tag{4.13}
$$

where the sum runs over the physical states $|\phi\rangle$ in sub-Hilbert space $\mathcal{H}_c$ of closed strings made with $\mathrm{SL}(2,\mathbb{R})$ continuous representations, that couple to the brane. In this equation $\tilde{j}$ is the SU(2) spin, n_i are the toroidal momenta, $D(N)$ is the density of states at oscillator number N and F the worldsheet fermion number for a given string state. All these labels have to be understood as functions of the state $|\phi\rangle$.[13] The term between square brackets is the contribution from $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ to the amplitude, while the last factor corresponds to the amplitude for the time-like boson part. The on-shell space-time energy E_a for a long string with w units of spectral flow and radial momentum P, in the NS ($a = 0$) or R ($a = 1$) sector, reads [33]:

$$
E_a = \frac{kw}{2} + \frac{2}{w}\left[\frac{P^2 + 1/4}{k} + \frac{\tilde{j}(\tilde{j}+1)}{k} + \frac{\sum_i(n_i/R_i)^2}{2} + N + \frac{a-1}{2}\right], \tag{4.14}
$$

Only left-right symmetric states couple to the brane. There are also no on-shell continuous representations for $w = 0$. The details of the computation are given in appendix B.

This quantity is related to the closed string emission by the brane decay. In terms of the coefficient $\mathcal{V}^w(E, P, \ldots)$ of the one-point function for a closed string vertex operator in

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[13]We should in principle take care more carefully of the different contributions to the density of states, in particular of the null states. However we will eventually be interested in the high-energy tail of the distribution for which these details are irrelevant.

the rolling tachyon background, eq. (3.9), it can be expressed as

$$\text{Im}\mathcal{A}_c = \frac{1}{2\pi} \sum_{|\phi\rangle} \int \frac{\mathrm{d}P}{\pi} \frac{1}{2} \sum_{a,b} \sum_{w\neq 0, N,\ldots} (-)^{bF} D(N) \frac{1}{|w|} \left| \langle \mathcal{V}^w \left(E_a\left[P, w, N, \ldots \right], P, \ldots \right) \rangle \right|^2$$

$$= \sum_{|\phi\rangle} \int \frac{\mathrm{d}E}{2\pi} \int_0^\infty \frac{\mathrm{d}P}{\pi} \frac{1}{2} \sum_{a,b} \sum_{w,N,\ldots} (-)^{bF} D(N) \times$$

$$\times \ \delta \left(-wE + \frac{kw^2}{2} + \frac{2(P^2 + 1/4)}{k} + 2N + \cdots \right) |\langle \mathcal{V}^w (E, P, \ldots) \rangle|^2 \,. \tag{4.15}$$

Following [23], this amplitude can be interpreted as $\bar{\mathcal{N}}_c$, the mean number of long strings produced by the D0-brane decay, traced over all the physical spectrum of closed strings in the continuous representations. We work in the gauge with no oscillators for the time-like boson, which is consistent as long as spacetime energy is not zero [76].[14] In the present case there are no physical long strings with zero energy. The invariant measure that appears in (4.15), with the on-shell constraint $\delta(L_0 + \bar{L}_0 + a - 1)$, is a natural generalization of the point-particle measure to string theory.

The absence of a delta-function representing the conservation of the *total* energy stored in the brane, which is justified for $g_s = 0$, is a consequence of the fact that eq. (4.15) is a tree-level amplitude (indeed one applies an optical-like theorem to the one-loop annulus amplitude).

High energy behavior. We would like now to check whether, like in flat space-time, there is an ultraviolet divergence associated with the production of very massive closed strings. The leading term in the density of states, for the left-right symmetric string states that couple to the D-brane, is given by [77, 78]:

$$\ln D(N) = 2\pi \sqrt{\frac{c_{\text{eff}} N}{6}} + \mathcal{O}(\ln N)\,, \tag{4.16}$$

where the effective central charge is $c_{\text{eff}} = 12 - 6/k$. To be more precise one should count the transverse physical degrees of freedom in space-time using the decomposition $\text{SL}(2, \mathbb{R}) \sim \text{SL}(2, \mathbb{R})/\text{U}(1) \times \mathbb{R}^{0,1}$. Then the effective central charge differs from the central charge by an amount proportional to the minimal scaling dimension in the spectrum of the coset. Alternatively the asymptotic degeneracy of states can be computed directly from the covariant AdS$_3$ partition function for the continuous representations, since their characters are identical to those of a free field theory, leading to the same result.[15] As in [34], the difference between the central charge and the effective central charge decreases the density of states at high energy compared to flat space-time, but in AdS$_3$ the main result is that there is no Hagedorn density of states in a given sector of spectral flow w.

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[14]Also the amplitude for a properly normalized descendant of the $\mathcal{N} = 2$ superconformal algebra in the $\text{SL}(2, \mathbb{R})/\text{U}(1)$ super-coset is the same, up to a phase, as the amplitude of the primary state. It can be shown using the (type B) boundary conditions on the $\mathcal{N} = 2$ SCA generators.

[15]Another way to reach this conclusion is to consider the effective linear dilaton CFT living on the long strings [79].

Indeed, the spacetime energy of long strings, in the spectral flow sector w, grows with the oscillator number as $E \sim 2N/w$. Therefore the suppression of the production of very energetic strings wins over the growth of the density of states which scales only like $\sqrt{N}$ as we saw above. It shows that, for a given sector of spectral flow, the production of very massive closed strings is exponentially suppressed.

Let us first look at the distributions of the closed string radiation for the radial momentum P and the oscillator number N, in a given sector of spectral flow w. In (4.13), the leading parts of the P-dependence from the $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ contribution cancel for large P, leaving an exponential contribution of the form $\exp(2\pi P/k)$. Omitting for the moment the summation over the zero modes of $\mathrm{S}^3 \times \mathrm{T}^4$, the amplitude (4.13) for large N and P behaves like:

$$\bar{\mathcal{N}}_c \sim \frac{1}{k} \sum_{w=1}^{\infty} \frac{1}{\omega} e^{-\pi\sqrt{\frac{k}{2}}w} \left\{ \int_0^\infty \mathrm{d}P \; e^{-\frac{2\pi}{k}\left[\sqrt{\frac{2}{k}}\frac{P^2}{w} - P\right]} \right\} \sum_N N^\gamma e^{-2\pi\left(\sqrt{\frac{2}{k}}\frac{N}{w} - \sqrt{(2-1/k)N}\right)} \quad (4.17)$$

Notice again the term in the second exponential proportional to N/w which differs from that similar term in flat space-time (or in the presence of a linear dilaton) where only $\sqrt{N}$ would have appeared. The exponent γ comes from the logarithmic corrections to the asymptotic density of states (4.16) [80]. The distributions in radial momentum P and oscillator number N for a given sector of spectral flow w are centered at

$$\bar{P}_w = \sqrt{\frac{k}{2}}\frac{|w|}{2} \quad , \qquad \bar{N}_w = \frac{w^2(2k-1)}{8}, \quad (4.18)$$

see figure 1. We see that the emitted long strings with large winding number are very excited; they have an energy of order $E \sim kw/2 + 2\bar{P}_w^2/(kw) + 2\bar{N}_w/w = kw$. The radius of a long string with radial momentum P and winding number w growths linearly in global AdS$_3$ time as [33]

$$\rho = \frac{2P}{k}\left|\frac{t}{w}\right|. \quad (4.19)$$

Plugging in this formula the mean value of P, eq. (4.18), we observe that the mean speed of growth of the long strings with large w emitted by the D0-brane decay is independent of their winding number, with a standard deviation of order $k^{-1/4}w^{-1/2}$.

To check whether the sum over the different sectors of spectral flow w gives a finite number of produced long strings, we first integrate (4.17) over P; indeed the term $\exp(2\pi P/k)$ can enhance the production of very massive long strings with large radial momentum. We get:

$$\bar{\mathcal{N}}_c \sim \sum_{w>0} \frac{1}{\sqrt{w}} e^{-\pi\sqrt{\frac{k}{2}}\left(1-\frac{1}{2k}\right)w} \sum_N N^\gamma e^{-2\pi\left(\sqrt{\frac{2}{k}}\frac{N}{w} - \sqrt{(2-1/k)N}\right)}. \quad (4.20)$$

Then in the large w limit one can replace the sum over the oscillator number N by an integral. We find that the exponential terms in w cancel from the amplitude (4.20). Therefore, the distribution of the radiation as a function of w is governed by the power-law corrections to the asymptotic density of states, much as in flat space-time.
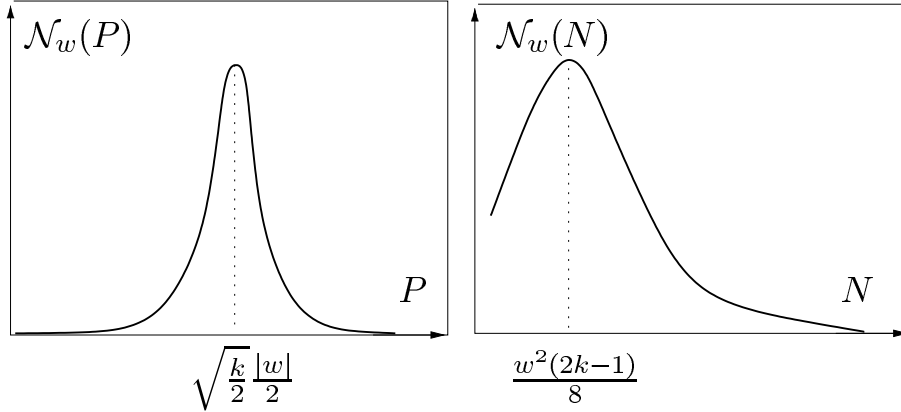
Figure 1: Distribution of the long strings radiation for the radial momentum P and the oscillator number N in a given sector of spectral flow w.

To evaluate the effect of the power-law corrections, we consider first the simplest case of a non-critical superstring $\mathrm{SL}(2,\mathbb{R})|_{k=2}\times\mathrm{S}^1\times\mathbb{R}^4$, with the brane localized on $\mathbb{R}^4$:

$$\bar{\mathcal{N}}_c \sim \sum_{w>0} \frac{1}{\sqrt{w}}\, e^{-\pi\left(1-\frac{1}{4}\right)w} \sum_N N^\gamma e^{-2\pi\left(\frac{N}{w}-\sqrt{3N/2}\right)} \left[\int \mathrm{d}p\; e^{-\pi\frac{p^2}{w}}\right]^4 \tag{4.21}$$

First, integrating over the $\mathbb{R}^4$ momenta one gets an extra factor of w^2 in the sum (4.21). Then, we have to count the contributions to the density of states of five free bosons and eight free fermions (two of them being the S^1 at the fermionic radius). Using the results of [81] we find a "universal" correction of order $N^{-3/4}$, and an extra contribution of order $N^{-5/4}$ from the five bosonic oscillators (in the light-cone gauge). Thus in this case the exponent of the power-law correction is $\gamma = -2$ and the sum (4.20) behaves like $\bar{\mathcal{N}}_c \sim \sum_{w>0} w^{-1}$. One can check that if one compactifies $\mathbb{R}^4$, for example on a square torus at the fermionic radius for which the computation is simple, this result holds.

We find that the total number of emitted long strings from the decaying D0-brane is log-divergent. Their mean energy is linearly divergent in w. It shows that the ultraviolet divergence of closed string emission is not removed in AdS$_3$, despite the much softer high-energy behavior.[16] Closed string perturbation theory breaks down due to a large backreaction from the long strings. The same conclusions can be reached for the other non-critical string backgrounds AdS$_3\times$ S$^1\times$ T^2 and AdS$_3\times$ S^1. In appendix C we discuss in detail the bosonic case where the states counting can be done explicitly. In the more generic cases like superstrings on AdS$_3\times$ S$^3\times$ T^4 the computation of the power-law corrections to the density of states is more involved but will very likely leads to the same conclusion. At least it is safe to say that, because of the power law behavior, higher moments $\overline{E^n}$ will be divergent for n large enough, signaling the breakdown of closed string perturbation theory.

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[16]Technically the factor $\sinh 2\pi P/k$ in (4.13) responsible for this result comes from the (absolute square of the) worldsheet non-perturbative corrections to the $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ one-point function, i.e. the term $\Gamma(1+\frac{2j-1}{k})$ in (3.9).

Non-perturbative regularization. The tree-level computation of closed strings radiation predicts that an infinite amount of energy is emitted by the decay of the D0-brane. This divergence is regularized once non-perturbative effects kick in. The spectral flow is non-perturbatively bounded from above, because long strings with winding number $w \sim Q_1$ carry an NS-NS two-form charge of the same order as the total charge of the Q_1 background fundamental strings, for an $AdS_3 \times S^3 \times T^4$ superstring with coupling constant $g_6 = \sqrt{k/Q_1}$. This regularization gives $\bar{\mathcal{N}}_c \sim -\log g_6^2$ and the average radiated energy as $\bar{E} \sim Q_1 \sim 1/g_6^2$. At weak coupling is far larger than the mass of the brane which is of order $1/g_6$.

One can understand this discrepancy, at least qualitatively, as follows. The amplitude contains contributions from long strings with energies larger than (in units of $1/\sqrt{\alpha'}$) $1/g_6$, the inverse six-dimensional string coupling. One may make the assumption that the contribution to the amplitude of these long strings, which lie outside the domain of validity of string perturbation theory, is at most subdominant (their energy is of order of, or larger than, the D0-brane mass). Therefore, since the typical energy of long strings with winding w is $E = kw$, to exclude strings with $E > 1/g_6$ one needs to put a cutoff of order $w < g_6^{-1} \propto \sqrt{Q_1}$ on the spectral flow number w. While from the point of view of the number of produced long strings the effect is not dramatic, it gives an average radiated energy of order $1/g_6$, in agreement with energy conservation. The distribution of closed strings radiation is peaked near the cutoff scale.

One might have expected a qualitative difference for $k < 1$,[17] because string theory on AdS_3 undergoes a phase transition there and long strings become weakly coupled near the boundary [79]. In our computation the leading approximation to closed strings production (i.e. the exponential terms) does not change at $k = 1$. The power-law corrections that determine the behavior of the system are model-dependent and would require a more detailed study.

Extended branes. The result found for long string emission seems peculiar to the D0-brane since it comes from the exponentially large contribution of states with large radial momentum (of order $e^{2\pi P/k}$) to the $SL(2,\mathbb{R})/U(1)$ part of the amplitude (4.13), see eq. (4.17). To obtain a UV finite long strings emission one may try to start with extended D-branes in AdS_3 (the analogue of an FZZT brane of Liouville theory).

We consider a symmetry-breaking D2-brane of AdS_3 in type IIB superstrings, made out of a D2-brane of the cigar. This D2-brane covers the whole AdS_3 space-time and has a magnetic field on its worldvolume; the latter is parameterized by a (quantized) label $\sigma \in [0, (1 + \frac{1}{k})\frac{\pi}{2})$ [43, 45].[18] The modulus squared of the one-point function for the D2-brane in $SL(2,\mathbb{R})/U(1)$ scales asymptotically with the radial momentum as $\exp\{\frac{2\pi P}{k}(\frac{2k\sigma}{\pi} - k - 1)\}$. Therefore, if one adds the rolling tachyon boundary deforma-

---

[17]For the $AdS_3 \times S^3 \times T^4$ background that we take as an example we have always $k \geqslant 2$ but the regime $k < 1$ can be obtained for certain non-critical backgrounds [79].

[18]The boundary CFT for the D2-brane in $SL(2,\mathbb{R})/U(1)$ is not completely well understood; see in particular [44, 46] for other proposals. We follow here the approach of [43, 45], continued to integer k [82]. Note that qualitatively the features discussed in this paragraph don't depend on the particular D2-brane boundary state chosen.

tion (3.3) to the AdS$_3$ worldsheet CFT with a D2-brane boundary, one finds that long string radiation is exponentially suppressed at high energies.

However, the physical interpretation of this worldsheet boundary CFT is very different from the D0-brane example. In the open string sector of the D2-brane, the identity (i.e. the NS open string vacuum) is not normalizable. The physical open string tachyon on the extended brane belongs to the continuous representations of SL(2,$\mathbb{R}$),with $j = 1/2 + iP$, $P \in \mathbb{R}_+$ and its wave-function is delta-function normalizable. Therefore one cannot interpret the worldsheet theory with the boundary deformation (3.3) as the decay of the unstable D2-brane. The latter would correspond to a boundary deformation built on continuous representations of SL(2,$\mathbb{R}$). We leave as an open problem the analysis of this more complicated worldsheet boundary CFT.

4.2.2 Short strings production

The closed string spectrum in AdS$_3$ contains also discrete representations in the range (2.3). They correspond to "short strings" trapped inside AdS$_3$ [33]. By following the same steps as above, we arrive to the following expression for the imaginary part of the annulus amplitude, that is interpreted as the mean number of emitted physical short strings:

$$\bar{\mathcal{N}}_d = \mathrm{Im}\,\mathcal{A}_d = \frac{1}{2k \prod_i R_i} \sum_{|\phi\rangle \in \mathcal{H}_d} \sum_{2\tilde{j}=1}^{k-2} \sin\left(\frac{\pi(2\tilde{j}+1)}{k}\right) \int_{\frac{1}{2}}^{\frac{k+1}{2}} \mathrm{d}j \sum_{\ell,w\in\mathbb{Z}} \sin\frac{\pi(2j-1)}{k} \times$$

$$\times \sum_{a,b} \sum_N (-)^{bF} D(N) \frac{(-)^a}{|2j-1+kw|\sinh^2\left(\frac{\pi E_a}{\sqrt{2k}} + \frac{i\pi a}{2}\right)} \times$$

$$\times \delta\left(j - \frac{1}{2} + \frac{k}{2}w - \sqrt{\frac{1}{4} + k\left[N + \frac{\tilde{j}(\tilde{j}+1)}{k} + \frac{\sum_i(\frac{n_i}{R_i})^2}{2} - w(\ell + \frac{a}{2}) - \frac{w+(1-a)}{2}\right]}\right), \quad (4.22)$$

summing over physical states belonging to the sub-Hilbert space $\mathcal{H}_d$ of closed strings made with discrete SL(2,$\mathbb{R}$) representations. The on-shell spacetime energy is now

$$E_a = 2\left(j + \ell + \frac{a}{2}\right) + kw \quad (4.23)$$

where j solves the delta-function constraint in eq. (4.22).

One of the main characteristics of the emitted short strings is that they don't go away far from the locus of the brane. Classical short strings with $w = 0$ are time-like geodesics with a periodic motion around the origin, while classical spectral-flowed short strings have a periodic breath mode [33]:

$$e^{i\phi}\sinh\rho = ie^{iw\sigma}\sinh\rho_0\sin\frac{2j\tau}{k} \quad \text{with} \quad \cosh\rho_0 = 1 + \frac{\ell}{j}, \quad (4.24)$$

where (τ,σ) are the worldsheet coordinates and (ρ, ϕ) the space-like global coordinates in AdS$_3$, see eq. (2.1). Unlike the continuous representations, there are on-shell discrete states in the sector $w = 0$. The contribution of short strings with $w = 0$ is UV finite since the upper bound on j of eq. (2.3) implies an upper bound on the oscillator number N in this sector.

Let us now examine the high energy behavior of the amplitude (4.22) in the sectors of non-zero spectral flow w. As for long strings, the density of states grows exponentially with the oscillator number N while the amplitude is exponentially suppressed as a function of the energy (4.23). Because of the bounds (2.3) on the $\mathrm{SL}(2,\mathbb{R})$ spin j, for fixed large spectral flow w, only the discrete states with $\ell \sim N/w - kw/4$ will contribute for $N \gg 1$. Therefore, from eq. (4.23) giving the space-time energy, the amplitude is weighted at large N by a factor

$$\frac{1}{\sinh^2 \frac{\pi E}{\sqrt{2k}}} \sim e^{-\pi\sqrt{\frac{k}{2}}w - 2\pi\sqrt{\frac{2}{k}}\frac{N}{w}}, \tag{4.25}$$

similar to what we obtained above for long strings, see eq. (4.20). It is interesting that we obtain, for short strings in a given spectral flow sector, the field theory entropy $S(E) \sim \sqrt{E}$ expected from AdS$_3$/CFT$_2$ duality [35] using the bound (2.3) and the on-shell condition.

The leading contribution to the asymptotic density of states for the transverse degrees of freedom, with a reasoning similar to the continuous representations, is given by (4.16) with the same $c_{\mathrm{eff}} = 12 - 6/k$.[19] In contrast with the long strings sector, the contribution of the square of the $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ one-point function coefficient to the annulus amplitude, see eq. (4.22), is of order one. The net effect is that the exponential suppression of short strings emission wins over the asymptotic density of states at high energy. Therefore the number of emitted short strings, evaluated at tree level, is UV finite. It means that, for a given AdS$_3$ radius (i.e. fixed k), one can choose the string coupling constant g_6 small enough in order to get an arbitrarily large fraction of the energy dissipated into long strings, rather than into short strings.

Infrared divergence. Because the spectrum of discrete $\mathrm{SL}(2,\mathbb{R})$ representations contains states with zero space-time energy, there is an infrared divergence in the computation of the mean number of emitted short strings. From the worldsheet CFT point of view it comes from the extra primaries at $E = 0$, that we discussed briefly in subsection 3.1. The only $\mathrm{SL}(2,\mathbb{R})$ physical states with zero energy (and compatible with the GSO projection) are the $\mathcal{N} = 2$ Liouville interaction [73] and its image under one unit of spectral flow. It is also worthwhile to remind that this divergence is eliminated if we choose the "full-S-brane" solution (3.1a) instead of the "half-S-brane" [23]. There is no indication that this infrared divergence is more harmful that in flat space-time.

Flat space-time limit. It is interesting to have a look at the $k \to \infty$ limit, in order to connect our results to those obtained in flat space-time.[20] Let's consider first the continuous $\mathrm{SL}(2,\mathbb{R})$ representations. Unflowed states ($w = 0$) are not part of the physical spectrum even for $k = \infty$ as they correspond to space-like geodesics. Long strings decouple in the flat space-time limit, because their mass is of order kw. The unflowed discrete representations play the main role in the $k \to \infty$ limit (the contributions of short strings with $w \neq 0$ to the amplitude are also suppressed in the large k limit). Indeed the upper bound on the

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[19]One can also find an upper bound on the degeneracy of states using the explicit expansion of the discrete representations characters, confirming this result.

[20]We thank Nissan Itzhaki for discussions about this issue.

$SL(2, \mathbb{R})$ spin j, see eq. (2.3), which prevents them from giving a significant contribution to closed string emission, disappears. At large N states with $j \sim \sqrt{kN}$ will give contributions to the amplitude (4.22) for closed string emission of order $e^{-2\pi\sqrt{2N}}$, like in flat space-time. As the density of states (4.16) converges also to the flat space-time value in the $k \to \infty$ limit, we recover the main aspects of the flat space-time results found in [24, 23]. In order to get a precise matching one needs to be very careful about the order of limits. The energy dissipated into short strings is independent of g_6 at leading order. For fixed g_6 if one increases k, i.e. decreases the AdS$_3$ curvature, one expects to reach a turnover point above which the short strings production is dominant.

Summary of closed string emission. Our computation of the annulus amplitude shows that, at small string coupling, almost all the energy radiated into closed strings resides in long strings with large winding number. The short strings production is finite and carries only a small fixed fraction of the energy. The radiation of long strings changes effectively the string coupling constant (in the interior of the shell of closed strings that are emitted), since $g_s^2 \propto 1/Q_1$, where Q_1 is the number of fundamental strings that build up the background. This quantity will of course receives higher order perturbative corrections, e.g. multi-closed strings emission which is of order $g_s^{2(N-1)}$ for N-particle states.

In flat space-time, the characteristics of the closed string radiation led Sen to the *open string completeness conjecture* [26], which states that the open string field theory description gives a complete description of the decay, in particular "takes care" of the apparently large back reaction due to massive closed strings. This open string description is approximated by the worldvolume Born-Infeld-like action for a non-BPS D-brane, which exhibits the distinctive features of tachyon condensation as described by closed string radiation, namely pressureless matter in the asymptotic future and no plane waves excitations near the minimum of the potential, signaling the presence of "tachyon dust" made of non-relativistic massive closed strings.

In the example studied here, D-particle decay in AdS$_3$, the radiation is made of macroscopic long strings. Both cases are attempts to take perturbation theory beyond its range of validity. The two pictures, flat space-time and AdS, coincide in the region where perturbation theory is valid, i.e. for a timescale of order $\sqrt{\alpha'}$, before the long strings radius become significantly larger than the string scale, and the two pictures diverge after that time.

4.3 Comparison with non-critical strings

As we demonstrated above, in AdS$_3$ the high-energy divergence in closed strings emission from the brane decay is regularized non-perturbatively. It has been suggested in [34] that for brane decay in non-critical strings the closed string emission is UV-finite a tree level, because the high-energy density of states is somewhat lower than in flat space-time. As anticipated in the introduction it is very unlikely that the high-energy behavior of non-critical strings could be softer than string theory in anti-de Sitter space-time. We would like to clarify this issue by uncovering similar divergences in non-critical strings (to those in AdS$_3$), although the mechanism of closed string emission is different.

For concreteness we consider unstable D-branes in superstring theories of the form $\mathbb{R}^{2n,1} \times \mathbb{R}_Q$. In order to obtain a stable supersymmetric background, and regularize the strong coupling region due to the linear dilaton, one has to compactify one of the spatial directions at a precise radius and add an $\mathcal{N} = 2$ Liouville potential. Then one obtains the worldsheet CFT $\mathbb{R}^{2n-1,1} \times \mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)|_{2/(4-n)}$ with the appropriate GSO projection.

As in [34] we could consider first a non-BPS brane extended along the linear dilaton direction ρ, i.e. made of a D1- or D2- brane of the cigar.[21] However, as for the D2-brane in AdS$_3$ discussed above, the rolling tachyon boundary deformation (3.3) is built with the identity of the $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ coset. The latter is not normalizable on the extended brane because of the asymptotic linear dilaton. It means that, while being a consistent worldsheet boundary CFT, it does not represent the decay of the physical open string tachyon living on the brane.

The open string tachyon on the extended brane belongs to the continuous representations of $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$, as for the extended brane in AdS$_3$ discussed above. To analyze its decay using BCFT methods one should consider instead the worldsheet theory deformed by a boundary marginal deformation of the asymptotic form

$$\delta S = \lambda \oint_{\partial \Sigma} \mathrm{d}\ell \; G_{-1/2} e^{-\frac{1/2+iP}{\sqrt{2k}}\rho + \sqrt{\frac{1}{4} - \frac{P^2+1/4}{2k}} X^0} \sigma^1 \,. \tag{4.26}$$

As in AdS we do not know how to solve the theory in the presence of this boundary term in the action even in the case $P = 0$, i.e. homogeneous decay. It would be interesting also to understand the physics of the rolling tachyon (3.3) in the non-critical strings context, in particular to understand if and why closed strings emission is finite there. In bosonic strings, similar statements can be made, considering an FZZT extended brane [64] of Liouville theory.[22]

The exact non-critical string analogue of the unstable D-particle that we study in AdS$_3$ is to consider a non-BPS D0-brane made with a D0-brane of the $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ coset, i.e. localized at the tip of the cigar (similar to a ZZ brane of bosonic Liouville theory). In the open string sector the identity of the $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ CFT is normalizable therefore one can describe the condensation of the physical open string tachyon with the boundary deformation (3.3). Let's consider six-dimensional superstrings, i.e. $\mathbb{R}^{5,1} \times \mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)|_{k=2}$ as an example. The one-point function for an operator in the the continuous representations of the coset is:

$$|\langle V^{\mathfrak{sl}_2/\mathfrak{u}_1}_{j=1/2+iP \, s\bar{s}} \, e^{iEX^0}\rangle|^2 \propto \delta_{s,-\bar{s}} \frac{\sinh 2\pi P \sinh \pi P}{\cosh 2\pi P + \cos \pi s} \frac{1}{\sinh^2 \pi E} \,, \tag{4.27}$$

where $(s, \bar{s})$ are the $\mathbb{Z}_4$-valued left and right momenta of $\mathrm{U}(1)_2$. In the non-critical superstring the on-shell condition is simply

$$E = \sqrt{P^2 + 1/4 + \frac{s^2}{2} + \mathbf{p}^2 + 2N - 1} \,, \tag{4.28}$$

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[21]The analogous problem of the decay of an FZZT brane in 2D bosonic string theory has been discussed in [83].

[22]In both cases, bosonic and supersymmetric, the bulk Liouville potential is not important in the discussion if one takes the boundary cosmological constant much larger than the bulk cosmological constant, in such a way that the brane "dissolves" in the weak coupling region.

where P is the radial momentum in $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ and $\mathbf{p}$ the $\mathbb{R}^5$ momentum transverse to the brane. The density of states that appears in the computation of the annulus amplitude, including the power-law corrections, is the same as our $\mathrm{SL}(2,\mathbb{R})|_{k=2} \times S^1 \times \mathbb{R}^4$ previous example:

$$D(N) \sim N^{-2} e^{2\pi\sqrt{\frac{3N}{2}}} \, . \tag{4.29}$$

So at large N and P the mean energy emitted by the brane decay scales like

$$\bar{E} \sim \int \mathrm{d}^5\mathbf{p} \int \mathrm{d}P \sum_N \frac{1}{N^2} \, e^{2\pi\sqrt{\frac{3N}{2}}} e^{\pi P} \, e^{-2\pi\sqrt{P^2 + 2N + \mathbf{p}^2}}$$

$$\sim \int \mathrm{d}P \sum_N \frac{1}{N^2} \, e^{2\pi\sqrt{\frac{3N}{2}}} \, e^{-\pi(2\sqrt{P^2+2N}-P)} \left[\int \mathrm{d}\rho \, e^{-\frac{\pi\rho^2}{\sqrt{P^2+2N}}} \right]^5 \, . \tag{4.30}$$

One can use a saddle point approximation to evaluate the integral over P at large N. As for the D0-branes in AdS_3, the exponential terms in $\sqrt{N}$ cancel, leaving the power-law corrections that determine the behavior of the amplitude. One obtains that the total energy radiated into closed strings, evaluated at tree level, diverges like $\sum_N N^{-1/2}$. Therefore in non-critical superstrings the UV divergence of closed strings emission remains.[23]

4.4 Open string pair production

One possible way of radiating the energy of the D-brane, as an intermediate stage of the brane decay, is to consider open string pair creation. It is not completely clear what is the meaning of this process since the open string loose their support. However if this emission happens to be divergent it singles the breakdown of open strings perturbation theory. This would render unreliable both the open string and the closed string computations at tree level.

Let's consider the boundary theory of an unstable D-particle in the presence of boundary deformation (3.1b). At past infinity $x^0 \to -\infty$ this interaction vanishes and the spectrum of open strings can be read from the D-particle partition function (2.11). A very important difference with the closed string spectrum is that the $\mathrm{SL}(2,\mathbb{R})$ contribution splits into an $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ part and a $\mathrm{U}(1)$ temporal part whose zero-modes are independent from each other. Therefore the scaling of the energy with the oscillator number is the same as in flat space: $E \sim \sqrt{N/\alpha'}$. However, despite the fact that the open string spectrum contains the identity, the effective central charge entering in the asymptotic density of states (4.16) is the same as in the closed string sector ($c_{\mathrm{eff}} = 12 - 6/k$), because this leading contribution is evaluated by modular transform of the annulus amplitude to the closed string channel where the minimal conformal dimension in the $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ CFT is $\Delta_{\min} = 1/4k$.

To conclude, AdS_3 string theory looses in the tree-level approximation one of its prominent features in the open string sector of the unstable D-particle. It has an Hagedorn growth of the density of states at high energies, with the same Hagedorn temperature as a

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[23]The decay of ZZ branes in 2D bosonic string theory has been considered in [84], and recently extended in [85] to higher dimensions. In all those cases the closed strings emission is also divergent.

non-critical superstring with a dilaton slope $Q = \sqrt{2/k}$.[24] This has to be contrasted with the spectrum of AdS_2 branes [29]. This fact may probably not be true non-perturbatively since string theory on AdS_3 is dual to a field theory and should not exhibit an Hagedorn growth of the density of states in any sector. In any case we will show that even with this Hagedorn behavior there is a finite average number of produced open strings because the Hagedorn temperature is higher than is flat space.

The computation of the amplitude for pair production of open strings uses the boundary two-point function, which plays the role of a Bogolioubov coefficient [19]. We have already seen that it is related to the energy density appearing in the open string annulus amplitude of eq. (4.10); we shall now discuss how to obtain its actual expression using the orbifold construction of subsection 3.3.

It is quite interesting that the open string annulus amplitude for the rolling tachyon, eq. (4.7), that we obtained by channel duality from the closed string annulus amplitude (4.1), does not exhibit poles corresponding to on-shell open strings pair-produced by the time-dependent interaction. As we saw above its imaginary part signals the production of closed strings. Some comments about this issue can be found in [34].

We consider in the following the bosonic case for technical convenience. The extension to the superstring, using the results of [69] is possible. We start with the boundary two-point function in ordinary Liouville theory (with $Q = b + b^{-1}$). The boundary two-point function, or reflection amplitude, for an open string "tachyon" of dimension $\Delta = \alpha(Q - \alpha)$, interpolating between the boundary conditions s_1 and s_2, is given by [64]:

$$d_b(\alpha|s_1, s_2) = \frac{b}{2\pi} \left(\mu\pi \frac{\Gamma(b^2)}{\Gamma(1-b^2)} \right)^{\frac{Q-2\alpha}{2b}} \Gamma\left(\frac{2\alpha}{b} - \frac{1}{b^2} \right) \Gamma\left(2b\alpha - b^2 - 1 \right) \times$$

$$\times \frac{S_b^{(0)}(2\alpha)}{S_b^{(0)}(\alpha + \frac{s_1+s_2}{2i}) S_b^{(0)}(\alpha - \frac{s_1+s_2}{2i}) S_b^{(0)}(\alpha + \frac{s_1-s_2}{2i}) S_b^{(0)}(\alpha - \frac{s_1-s_2}{2i})} , \quad (4.31)$$

using the special function $S_b^{(0)}(x)$ defined by eq. (4.6). First, as discussed in subsection 3.1, we are interested in the limit of vanishing Liouville interaction $\mu \int d^2z \exp 2b\rho(z, \bar{z})$, while keeping fixed the boundary cosmological constant $\tilde{\lambda}$, i.e. with a fixed boundary interaction of the form:

$$\delta S = \frac{\tilde{\lambda}}{2} \oint_{\partial\Sigma} d\ell \, e^{b\rho(\ell)} . \quad (4.32)$$

This regime corresponds to a brane which "dissolves" way before the Liouville potential becomes important because the boundary potential (4.32) acts like a barrier for open string modes. The boundary parameter s is related to the boundary cosmological constant as

$$\tilde{\lambda}^2 = \frac{4\mu}{\sin \pi b^2} \cosh^2(\pi bs) , \quad (4.33)$$

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[24]Of course since this D-brane is unstable the thermodynamics of open strings attached to it is not well-defined.

such that we consider the limit $\mu \to 0$, $s_{1,2} \to \infty$, with $(s_1 - s_2)$ fixed. As in [19], using the asymptotics of $S_b^{(0)}(x)$ one gets for the boundary two-point function:

$$d_b(\alpha|s_1, s_2) = \frac{b}{2\pi} \left(\frac{\pi^2 \tilde{\lambda}_1 \tilde{\lambda}_2}{\Gamma(1 - b^2)^2} \right)^{\frac{Q - 2\alpha}{2b}} \Gamma\left(\frac{2\alpha}{b} - \frac{1}{b^2} \right) \Gamma\left(2b\alpha - b^2 - 1 \right) \times \qquad (4.34)$$

$$\times \frac{S_b^{(0)}(2\alpha)}{S_b^{(0)}(\alpha + \frac{s_1 - s_2}{2i}) S_b^{(0)}(\alpha - \frac{s_1 - s_2}{2i})} . \qquad (4.35)$$

The boundary two-point function for time-like boundary Liouville theory is obtained in the limit $b \to i$. However in this limit the function $S_b^{(0)}(x)$ has an infinite number of poles accumulating for every $x \in i\mathbb{Z}$. One therefore should give some prescription for this limit; we refer the reader to [19, 66] for details about these issues.

In the orbifold construction of the rolling tachyon in AdS$_3$, see subsection 3.3, we start with branes on the covering space of the orbifold, i.e. SL$(2, \mathbb{R})/$U$(1)_\infty^{\vee} \times \mathbb{R}^{0,1}$, and sum over the images under the geometric identification. In the orbifold theory there are new boundary operators corresponding to open strings stretched between the brane and one of its images.[25]

Open strings in the rolling tachyon BCFT stretched between a brane and its image under the translation $\mathfrak{T}^r$ correspond to boundary operators interpolating between the boundary cosmological constants[26] $\tilde{\lambda}_1$ and $\tilde{\lambda}_2 = \tilde{\lambda}_1 e^{2\pi r/\sqrt{k}}$, i.e. the temporal part of the boundary two point function is given by the $b \to i$ limit of (4.35) with $s_1 - s_2 = 2ir/\sqrt{k}$. The annulus amplitude for open strings stretched between two such branes contains only the coset character $ch_{\mathbb{I}}(r; \tau)$ coming from the identity representation of $\widehat{\mathfrak{sl}}(2, \mathbb{R})$, see the open string partition function (4.7).

Using the prescription of [19] one finds that the temporal part of the boundary two-point function in AdS$_3$, for an open string vertex operator of the form $e^{iv X^0}$ of conformal dimension $\Delta = -v^2$ is given by:[27]

$$d_i(v|r) = \left(\pi \tilde{\lambda} e^{\frac{\pi r}{\sqrt{k}}} \right)^{2iv} \frac{S_1^{(0)}(1 - 2iv)}{S_1^{(0)}\left(1 - iv - \frac{ir}{\sqrt{k}} \right) S_1^{(0)}\left(1 - iv + \frac{ir}{\sqrt{k}} \right)} \times$$

$$\times \frac{\sinh \pi(v + \frac{r}{\sqrt{k}}) \sinh \pi(v - \frac{r}{\sqrt{k}})}{\sinh^2 2\pi v} . \qquad (4.36)$$

This result is as expected closely related to the (bosonic version of the) energy density (4.10). Using the identity $S_b^{(0)}(x) S_b^{(0)}(Q_b - x) = 1$ we observe that the first line of the boundary two-point function (4.36) is a pure phase.

The SL$(2, \mathbb{R})/$U(1) part of the boundary operator belongs to the coset module built on the SL$(2, \mathbb{R})$ operator in the identity representation with $J^3 = r$, see the orbifold action (3.17) and the partition function (4.7), and can be normalized to one. Therefore

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[25] See the reference [86] for a detailed analysis of the rational CFT analogue.

[26] There is a factor of $\sqrt{2}$ in the bosonic theory compared to the superconformal case discussed in subsection 3.3.

[27] The methods developed in [66] are more appropriate to the Euclidean $c = 1$ Liouville theory.

the modulus of the boundary two-point function for the rolling tachyon in AdS$_3$ in the r-sector is:

$$|d(\upsilon|r)_{\text{AdS}}| = \left| \frac{\sinh \pi(\upsilon + \frac{r}{\sqrt{k}}) \sinh \pi(\upsilon - \frac{r}{\sqrt{k}})}{\sinh^2 2\pi\upsilon} \right| . \tag{4.37}$$

Following [19] one can identify the boundary reflection amplitude $d(\upsilon|r)$ with the Bogolioubov coefficient $-\gamma_\upsilon^{\text{out}*} = \beta_\upsilon/\alpha_\upsilon$. Then the vacuum amplitude giving the rate of pair production (see [34] for details) is obtained as:

$$\mathcal{W} = -\text{Re}\,\ln\langle\text{out}|\text{in}\rangle = -\frac{1}{4}\sum_{N,r} D(N,r)\ln(1 - |\gamma_\upsilon(N,r)|^2)$$

$$= -\frac{1}{4}\sum_{N,r} D(N,r)\ln\left(1 - |d(\upsilon(N,r)|r)_{\text{AdS}}|^2\right) . \tag{4.38}$$

The on-shell energy for an open string in the r-sector is:

$$\upsilon(N,r) = \sqrt{\frac{r^2}{k} + r + N - 1} \tag{4.39}$$

Using the asymptotics of (4.37) for large υ and the high energy density of open string states as discussed above, we find the UV asymptotic behavior

$$\mathcal{W} \sim \sum_{r\in\mathbb{Z}}\sum_N e^{-4\pi\sqrt{\frac{r^2}{k}+r+N-1}\left(1-\sqrt{1-\frac{1}{4k}}\right)} . \tag{4.40}$$

Therefore open string pair production from the D0-brane decay in AdS$_3$ is exponentially convergent at high energies because, as in [34] for non-critical strings, the high-energy density of states is lowered compared to flat space-time. The same conclusion holds for the superstring case. It shows that open string perturbation is not invalidated by an infinite open string pair production. The average number of emitted open string pairs does not depend on the string coupling g_s in the leading order, thus open string pair production is negligible compared to the emission of closed long strings at weak coupling. In the $k \to +\infty$ flat space-time limit, the open string pair production is governed by power-law corrections and its divergences depend on the dimensionality of the brane and the specific energy moment considered.

5. Comments on holography

One of the main reasons to study string theory on AdS$_3$ is that, besides being an interesting example of curved space-time, it is an incarnation of the AdS/CFT holographic correspondence between gravity and field theory [35]. It is actually the only example at our disposal for which the string theory is under control, at least in the perturbative regime, because of its realization as an $\text{SL}(2,\mathbb{R})$ WZW model.

However life is not so simple; the space-time two-dimensional CFT dual to AdS$_3$ backgrounds with only NS-NS flux (i.e. with a WZW model description) is *singular* because the brane configuration that realizes this field theory on their worldvolume in the infrared limit

can fragment at no cost of energy [32]. One can move away from this singularity by turning on moduli in the space-time CFT. They correspond in space-time to Ramond-Ramond fluxes that invalidates the RNS construction of the worldsheet theory. As these fluxes prevent long strings from expanding towards the boundary, it modifies quite drastically the physics of the decay. We refer the reader to the review [87] for more details and references.

Once focusing on the BPS sector of the space-time CFT one can argue that certain quantities are protected by supersymmetry, allowing to compare results from the supergravity/worldsheet CFT side and from the space-time CFT at a non-singular point in the moduli space [88 – 90]. We are interested here in the decay of a non-BPS state, materialized on the gravity side as a D-particle, therefore there is no simple way to find its dual in the space-time theory. We will collect in this section some facts that may hint the solution of this problem.

The D1-D5 system and the symmetric product CFT. Among the various AdS$_3$ backgrounds, type IIB on AdS$_3\times$ S$^3\times$ T^4 has the most studied holographic dual; indeed it is related to the microscopic construction of supersymmetric black holes. This space-time is obtained as the near-horizon limit of k NS5-branes wrapped on T^4 and Q_1 fundamental strings smeared on the compact manifold.

In the S-dual picture, i.e. as a collection of D1- and D5-branes, the open string massless degrees of freedom give a two-dimensional U(k)$\times$U(Q_1) quiver gauge theory with $\mathcal{N} = (4,4)$ supersymmetry. At very low energies, the Higgs branch of the quiver theory flows to an $\mathcal{N} = (4,4)$ superconformal theory, a non-linear sigma model for the hypermultiplets whose target space is an hyper-Kähler manifold, together with a free $\mathcal{N} = (4,4)$ CFT corresponding to the center of mass coordinates on T^4. The target space of the former can be seen as the moduli space $\mathcal{M}$ of Q_1 instantons in a U(k) gauge theory on T^4. This SCFT lies in the moduli space of the symmetric product $(T^4)^{kQ_1}/S(kQ_1)$ and has a central charge $c = 6kQ_1$ [91]. For the "pure" D1/D5-system – or equivalently via S-duality the NS5/F1 background without Ramond-Ramond fluxes – this conformal field theory is singular, because of the small instanton singularities in $\mathcal{M}$. From the 1+1 gauge theory perspective it is obtained by turning off the Fayet-Iliopoulos (FI) terms and the theta-angle. This process corresponds to a situation where a long D-string, viewed as an instanton in the D5-brane worldvolume, "leaves" the system of branes [32]. The singularity occurs where classically the Higgs branch and the Coulomb branch meet.

5.1 Instantons, sphalerons in AdS$_3$ and their holographic dual

Before addressing the difficult problem of finding the dual of the unstable D-particle, we would like to understand precisely the meaning of the AdS$_3$ D-instanton constructed in section 2 in the context of the space-time CFT. These two objects are closely related for two reasons.

First, the D-instanton and the D-particle boundary states in AdS$_3$ differ only by the gluing conditions along the time direction; they come from the same D-brane in the coset SL$(2,\mathbb{R})$/U(1). Moreover, the worldsheet analysis of the D0-brane decay involves also D-instantons in an interesting way. Following [23], we can interpret the imaginary part of the

annulus diagram for the rolling tachyon, eq. (4.12), as a sum over contributions from an array of D-instantons (whose annulus amplitude is given in [30]) along the imaginary time axis.

Second, and more importantly, it has been argued [92] that the unstable D0-brane in type IIB superstring theory is a sphaleron, i.e. an unstable classical solution associated with a non-contractible loop in configuration space [93]. Inspired by this idea, it has been shown that in $\text{AdS}_5 \times \text{S}^5$ the D-particle located at the origin of global coordinates in AdS_5 is indeed a sphaleron in the gauge theory [37]. The latter is related to the SYM instanton in the sense that it is a classical solution of theory at the maximum of the potential barrier between two vacua of the gauge theory for which the instanton represents the tunneling process. It allowed to argue that the sphaleron survives at strong coupling – which is the regime where the holographic duality can be probed on the string side – because the associated instanton is a topologically stable and BPS object.

Instantons in the D1/D5 system. As discussed above the worldvolume theory of the D1/D5 system is described in the infrared limit, dual to string theory on $\text{AdS}_3 \times \text{S}^3 \times \text{T}^4$, by a non-linear sigma model on $\mathcal{M}$ with $\mathcal{N} = (4,4)$ superconformal symmetry. Near the point where one D1-brane could leave the system (for which a Coulomb branch opens up) the dynamics of the system is well described by a non-linear sigma model on the cotangent bundle of $\mathbb{CP}^k$. Then one gets instantons associated with maps $\mathbb{C} \to \mathbb{CP}^k$ described as vortices in the linear sigma-model description of the theory [94] (see [95, 96] for recent works on this subject). The action of these BPS instantons vanishes for zero FI terms, i.e. when the singularities are blown-down, and zero theta-angle. The AdS/CFT dictionary matches the FI terms with the self-dual part of the NS-NS two-form on T^4, in the D1/D5 description. One may identify these instantons with the Euclidean D1-branes wrapping holomorphic cycles of T^4 discussed in [97]. With an NS-NS two-form turned on, a D1-charge is induced in the presence of a non-zero instanton charge density through the coupling $\int_{\mathbb{R} \times S^1 \times T^4} B \wedge C_2 \wedge F$ on the D5-brane worldvolume.

It is however not clear to us what is the dual of the D(-1)brane, that we constructed in the $\text{AdS}_3 \times \text{S}^3 \times \text{T}^4$ background with only NS-NS fluxes (i.e. at the singular point) since a D(-1)-D1-D5 configuration is not supersymmetric.[28] The D(-1)-brane is expected to form a BPS bound state with the D1/D5 system "dissolving" in their wordvolume. As for the D0-D2 bound state, the D(-1) turns into an electric field in the D1-brane worldsheet, inducing a D(-1) charge through the coupling $\int_{\mathbb{R} \times S^1} C_0 F$ in the D-string action. Similarly an Euclidean D3-brane wrapped on the T^4 can dissolve in the D5-branes as an electric field.[29] In the AdS/CFT correspondence the theta angle of the D1/D5 gauge theory is identified with a linear combination of the RR axion C_0 and the RR four-form C_4 on T^4. Since the theta-angle in two dimensions is equivalent to an electric field, it may be related to (a superposition of) D(-1) and D3 instantons.

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[28]We would like to thank O. Aharony, A. Mikhailov, S. Rey and D. Tong for discussions about this problem.

[29]In the S-dual NS5/F1 description, these two kinds of D-instantons are both point-like in the AdS_3 space-time and differ only by the boundary conditions (Dirichlet or Neumann) on T^4.

An amusing observation about these D-instantons comes from the boundary worldsheet CFT solution itself. To find what kind of object the D-instanton represents in the space-time CFT it is convenient to Fourier transform the worldsheet vertex operators of the Euclidean AdS$_3$ (i.e. H$_3^+$) CFT, in target Euclidean space-time. In this basis the primary operators are written as $\Phi^j(u, \bar{u}|z, \bar{z})$, where $(u, \bar{u})$ are the coordinates on the plane where the *space-time* CFT lives, whereas $(z, \bar{z})$ are the worldsheet coordinates. Then the one-point function in the presence of the D-instanton takes the form [28]

$$\langle \Phi^j(u, \bar{u}|z, \bar{z}) \rangle = \frac{1}{|z - \bar{z}|^{2\Delta}} \frac{U(j)}{(1 + u\bar{u})^{2j}} \, . \tag{5.1}$$

The $(z, \bar{z})$ dependence on the upper-half plane is fixed by conformal symmetry on the worldsheet, while the $(u, \bar{u})$ dependence is fixed by conformal symmetry on the boundary of space-time where the dual space-time CFT is defined. It is very surprising that this functional form gives exactly a one-point function for an operator of dimension $\Delta_{\text{ST}} = \bar{\Delta}_{\text{ST}} = j$ in the space-time CFT on $\mathbb{RP}_2$, i.e. in the presence of a *crosscap*. It leads us to the rather weird conclusion that, from the point of view of the space-time theory, the D-instanton corresponds to a crosscap.[30] We will not push forward this interpretation because, while suggested by the boundary worldsheet CFT computation, its meaning is not very clear to us.

As for the D-instantons, the very existence of the non-BPS D0-brane in the string theory suggests that there is a corresponding sphaleron in the space-time CFT. It would be very interesting to understand how its decay could be related to the holographic dual of long strings emission, i.e. the passage from a Higgs to a Coulomb branch corresponding to the partial fragmentation of the brane stack.

Acknowledgments

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[30]One fact supporting this interpretation is that the open string sector of the D-instanton contains only the vacuum representation of SL(2, $\mathbb{R}$) (i.e. of spin $j = 0$), mapped to an operator of dimension zero in the space-time CFT; it is consistent with the fact that there are no degrees of freedom associated with an orientifold plane.

A. AdS$_3$ conformal field theory and characters

In this appendix we recall some facts about string theory on AdS$_3$ that are used in the body of the text, with an emphasis on the analytic continuation. We are interested in the SL$(2, \mathbb{R})$ super-WZW at level k, made with a purely bosonic WZW model at level $k+2$ and three free fermions of signature $(-, +, +)$.

SL$(2, \mathbb{R})$ from SL$(2, \mathbb{R})/$U(1). To deal with the Lorentzian signature of the SL$(2, \mathbb{R})$ group manifold, it is convenient to decompose the SL$(2, \mathbb{R})$ representations according to its time-like elliptic subalgebra. It corresponds to the equivalence

$$\mathrm{SL}(2, \mathbb{R})_k \sim \frac{\mathrm{SL}(2, \mathbb{R})/\mathrm{U}(1)|_k \times \mathrm{U}(1)_{-k}}{\mathbb{Z}_k} \,, \tag{A.1}$$

where the right-hand side is written in terms of the super-coset SL$(2, \mathbb{R})/$U(1) and a time-like free boson X^0 at radius $\sqrt{2k}$. AdS$_3$ spacetime is the universal cover of the SL$(2, \mathbb{R})$ group manifold, obtained by taking a continuous $\mathbb{Z}$ orbifold instead of the discrete $\mathbb{Z}_k$. One can use this decomposition to define the Wick rotation of the AdS$_3$ CFT in moduli space, see [30] for details.

To construct the closed string spectrum of the theory, we start with the partition function of SL$(2, \mathbb{R})/$U$(1) \times$U(1) and implement the diagonal orbifold action that defines the Euclidean AdS$_3$ CFT in the standard way compatible with modular invariance. The conformal weights of the SL$(2, \mathbb{R})/$U(1) primaries in the NS-NS sector are given by

$$L_0 = -\frac{j(j-1)}{k-2} + \frac{(n+kw)^2}{4k} \quad , \qquad \bar{L}_0 = -\frac{j(j-1)}{k-2} + \frac{(n-kw)^2}{4k} \,. \tag{A.2}$$

n and w are respectively the momentum and winding around the cigar at infinity. The corresponding vertex operators are written as

$$V_{\frac{n-kw}{2}, \frac{n+kw}{2}}^{j, \, \mathfrak{sl}_2/\mathfrak{u}_1}(z, \bar{z}) \tag{A.3}$$

The vertex operators of the SL$(2, \mathbb{R})$ WZW are represented in the orbifold theory (A.1) as follows:

$$V_{m \bar{m} w_+}^{j, \, \mathfrak{sl}_2} = V_{m - \frac{kw_+}{2}; -\bar{m} + \frac{kw_+}{2}}^{j, \, \mathfrak{sl}_2/\mathfrak{u}_1} e^{i\sqrt{\frac{2}{k}}(mX^0(z) + \bar{m}X^0(\bar{z}))} \,. \tag{A.4}$$

with $m, \bar{m} \in \mathbb{R}$ and $m - \bar{m} = n$. The sum w^+ of the winding numbers of the SL$(2, \mathbb{R})/$U(1) and U(1) theories is identified with the sector of *spectral flow*.

Representations and characters. The characters of the SL$(2, \mathbb{R})/$U(1) super-coset at level k come in different categories corresponding to the classes of irreducible representations of the SL$(2, \mathbb{R})$ algebra in the parent theory. In all cases the quadratic Casimir of the representations is $c_2 = -j(j-1)$.

Firstly we consider *continuous representations*, with $j = 1/2 + ip$, $p \in \mathbb{R}^+$. The characters are denoted by $ch_c(p, m)\begin{bmatrix} a \\ b \end{bmatrix}$, where the $N = 2$ superconformal U$(1)_R$ charge of the primary is $Q = 2m/k$, $m \in \mathbb{Z}/2$. Explicitly they are given by:

$$ch_c(p, m; \tau, \nu)\begin{bmatrix} a \\ b \end{bmatrix} = q^{\frac{p^2 + m^2}{k}} e^{4i\pi\nu\frac{m}{k}} \frac{\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(\tau, \nu)}{\eta^3(\tau)} \,. \tag{A.5}$$

Then we have *discrete representations* with $1/2 \leqslant j \leqslant {k+1}/2$, of characters $ch_d(j,r)\begin{bmatrix}a\\b\end{bmatrix}$, where the $N=2$ $U(1)_R$ charge is $Q = (2j+2r+a)/k$, $r \in \mathbb{Z}$. The characters read:

$$ch_d(j,r;\tau,\nu)\begin{bmatrix}a\\b\end{bmatrix} = q^{\frac{-(j-1/2)^2+(j+r+a/2)^2}{k}} e^{2i\pi\nu\frac{2j+2r+a}{k}} \frac{1}{1+(-)^b e^{2i\pi\nu}q^{1/2+r+a/2}} \frac{\vartheta\begin{bmatrix}a\\b\end{bmatrix}(\tau,\nu)}{\eta^3(\tau)}.$$
(A.6)

While the closed string spectrum in $SL(2,\mathbb{R})$ contains only discrete and continuous representations, the spectrum of open string attached to localized D-branes is built on the *identity representation*. The character for this identity representation we denote by $ch_{\mathbb{I}}(r)\begin{bmatrix}a\\b\end{bmatrix}$. It is given by:

$$ch_{\mathbb{I}}(r;\tau,\nu)\begin{bmatrix}a\\b\end{bmatrix} = \frac{(1-q)\,q^{\frac{-1/4+(r+a/2)^2}{k}}e^{2i\pi\nu\frac{2r+a}{k}}}{\left(1+(-)^b e^{2i\pi\nu}q^{1/2+r+a/2}\right)\left(1+(-)^b e^{-2i\pi\nu}q^{1/2-r-a/2}\right)} \frac{\vartheta\begin{bmatrix}a\\b\end{bmatrix}(\tau,\nu)}{\eta^3(\tau)}. \quad \text{(A.7)}$$

These characters have the reflection symmetry

$$ch_{\mathbb{I}}\begin{bmatrix}-a\\-b\end{bmatrix}(-r;it) = ch_{\mathbb{I}}\begin{bmatrix}a\\b\end{bmatrix}(r;it). \tag{A.8}$$

The primaries in the NS sector for this identity representation are as follows. First we have the identity operator $|j=0,r=0\rangle \otimes |0\rangle_{\text{NS}}$. The other primary states are:

$$|r\rangle = \psi^+_{-\frac{1}{2}}|0\rangle_{\text{NS}} \otimes (J^+_{-1})^{r-1}|0,0\rangle_{\text{BOS}} \quad \text{for } r>0 \quad \text{with} \quad L_0 = \frac{r^2}{k} + r - \frac{1}{2}$$

$$|r\rangle = \psi^-_{-\frac{1}{2}}|0\rangle_{\text{NS}} \otimes (J^+_{-1})^{-r-1}|0,0\rangle_{\text{BOS}} \quad \text{for } r<0 \quad \text{with} \quad L_0 = \frac{r^2}{k} - r - \frac{1}{2}.$$

B. From the annulus to closed strings emission

In this appendix we show how to get the mean number of emitted closed strings from the imaginary part of the annulus amplitude, given by eqn, (4.12). As quoted in the text the sum over r is finite because the divergent terms from $\exp -\pi t(n - i\sqrt{2/k}(r + b/2))^2$ are compensated with the weights in the $SL(2,\mathbb{R})/U(1)$ characters. However it prevents from performing the modular transform to the closed string channel in a straightforward way. To do so we need to add a regulator to the amplitude (4.12):

$$\text{Im}\,\mathcal{A}_\epsilon = \frac{1}{2} \int_0^\infty \frac{dt}{2t} \sum_{a\in\mathbb{Z}_2} (-)^b \chi^0(it) \frac{\vartheta\begin{bmatrix}b\\a\end{bmatrix}^3(it)}{\eta^3(it)} \sum_{\{w^i\}} \frac{e^{-\pi t(R_i w^i)^2}}{\eta^3(it)} \sum_{r\in\mathbb{Z}} e^{-\pi\epsilon r^2}\, ch_{\mathbb{I}}\begin{bmatrix}b\\a\end{bmatrix}(r;it) \times$$

$$\times \sum_{n=1}^\infty (-)^{a(n+1)}\, n\, e^{-\pi t\left(n-i\sqrt{\frac{2}{k}}(r+b/2)\right)^2}. \tag{B.1}$$

Then we can modular transform the various factors and get first a contribution from the

continuous representations, with $s = 1/t$:

$$\frac{\pi}{k^2 \prod_i R_i} \int_0^\infty \mathrm{d}s \sum_{a \in \mathbb{Z}_2} (-)^b \sum_{\tilde{j}} \sin \frac{\pi(2\tilde{j}+1)}{k} \chi^{\tilde{j}}(is) \frac{\vartheta \begin{bmatrix} a \\ b \end{bmatrix}^3 (is)}{\eta^3(is)} \sum_{\{n_i\}} \frac{e^{-\pi s (n_i/R_i)^2}}{\eta^3(is)} \times$$

$$\times \int \mathrm{d}P \mathrm{d}\mu \frac{\sinh 2\pi P \sinh 2\pi P/k}{\cosh 2\pi P + \cos 2\pi\mu} ch_c(P,\mu;is) \sum_{r \in \mathbb{Z}} e^{-\pi \epsilon r^2} e^{-\frac{4i\pi\mu(r+b/2)}{k}} \times$$

$$\times \sum_{n=1}^\infty (-)^{a(n+1)} \int \mathrm{d}p_0 \, e^{-\frac{\pi s p_0^2}{2k}} e^{2i\pi p_0 (\frac{n}{\sqrt{2k}} - \frac{i(r+b/2)}{k})} . \qquad (B.2)$$

Now we expand the contribution of the various characters in the partition function. We have schematically, e.g. in the NS sector:

$$\chi^{\tilde{j}}(is) \frac{\vartheta \begin{bmatrix} 0 \\ b \end{bmatrix}^3 (is)}{\eta^3(is)} \frac{e^{-\pi s \sum_i (n_i/R_i)^2}}{\eta^3(is)} \, ch_c \begin{bmatrix} 0 \\ b \end{bmatrix} (P,\mu;is)$$

$$\sim \sum_{N;\,\mathrm{NS}} (-)^{bF} D(N) e^{-2\pi s \left(\frac{\tilde{j}(\tilde{j}+1)}{k} + \frac{P^2+1/4}{k} + \frac{\mu^2}{k} + \frac{(n_i/R_i)^2}{2} + N - \frac{1}{2} \right)} . \qquad (B.3)$$

where $D(N)$ is the density of states at oscillator number N, and F the worldsheet fermion number for a given string state. Of course one would need to expand more rigorously the various characters and to keep track of the null vectors of $SU(2)_k$. After integrating over the Schwinger parameter s we get

$$\frac{1}{2k^2 \prod_i R_i} \sum_{a,b} \sum_N \sum_{\tilde{j}} \sin \frac{\pi(2\tilde{j}+1)}{k} \sum_{n=1}^\infty n(-)^{a(n+1)+bF} D(N) \times$$

$$\times \int \mathrm{d}\mu \mathrm{d}P \frac{\sinh 2\pi P \sinh 2\pi P/k}{\cosh 2\pi P + \cos 2\pi\mu} e^{-\frac{2i\pi\mu b}{k}} \times \qquad (B.4)$$

$$\int \mathrm{d}p_0 \frac{e^{2i\pi p_0 (\frac{n}{\sqrt{2k}} + \frac{ib}{2k})}}{\frac{p_0^2}{4k} + \frac{\mu^2}{k} + \frac{\tilde{j}(\tilde{j}+1)}{k} + \frac{P^2+1/4}{k} + \frac{\mu^2}{k} + \frac{(n_i/R_i)^2}{2} + N - \frac{a-1}{2}} \sum_{r \in \mathbb{Z}} e^{-\pi \epsilon r^2 + 2\pi p_0 \frac{r}{k} - \frac{4i\pi\mu r}{k}}$$

The integral over p_0 closes in the upper-half plane and gets a contribution from the simple pole at $p_0 = 2i\sqrt{\mu^2 + k\Delta}$ with

$$\Delta = \frac{\tilde{j}(\tilde{j}+1)}{k} + \frac{P^2 + 1/4}{k} + \frac{\mu^2}{k} + \frac{(n_i/R_i)^2}{2} + N + \frac{a-1}{2} . \qquad (B.5)$$

We get

$$\frac{\pi}{k \prod_i R_i} \sum_{a,b} \sum_N \sum_{\tilde{j}} \sin \frac{\pi(2\tilde{j}+1)}{k} \sum_{n=1}^\infty n(-)^{a(n+1)+bF} D(N) \times$$

$$\times \int \mathrm{d}\mu \mathrm{d}P \frac{\sinh 2\pi P \sinh 2\pi P/k}{\cosh 2\pi P + \cos 2\pi\mu} e^{-\frac{2i\pi\mu b}{k}} \times$$

$$\times \int \mathrm{d}p_0 \frac{e^{-4\pi \sqrt{\mu^2 + k\Delta}(\frac{n}{\sqrt{2k}} + \frac{ib}{2k})}}{\sqrt{\mu^2 + k\Delta}} \sum_{r \in \mathbb{Z}} e^{-\pi \epsilon r^2} e^{\frac{4i\pi r}{k}(\sqrt{\mu^2 + k\Delta} - \mu)} . \qquad (B.6)$$

Now we can take the limit $\epsilon \to 0$ and rewrite this as

$$\frac{\pi}{2\prod_i R_i} \sum_{a,b} \sum_N \sum_{\tilde{\jmath}} \ \sin\frac{\pi(2\tilde{\jmath}+1)}{k} \sum_{n=1}^\infty n(-)^{a(n+1)+bF} D(N) \times$$

$$\times \int \mathrm{d}\mu \mathrm{d}P \frac{\sinh 2\pi P \sinh 2\pi P/k}{\cosh 2\pi P + \cos 2\pi\mu} e^{-2i\pi b(\mu - \sqrt{\mu^2 + k\Delta})} \ \frac{e^{-4\pi\sqrt{\mu^2 + k\Delta}\frac{n}{\sqrt{2k}}}}{\sqrt{\mu^2 + k\Delta}} \times$$

$$\times \sum_{w\in\mathbb{Z}} \delta(\mu - \sqrt{\mu^2 + k\Delta} + \frac{kw}{2}). \tag{B.7}$$

And finally integrating over μ gives

$$\frac{\pi}{k\prod_i R_i} \sum_{a,b} \sum_N \sum_{\tilde{\jmath}} \ \sin\frac{\pi(2\tilde{\jmath}+1)}{k}(-)^{a+bF} D(N) \times$$

$$\times \sum_{w\neq 0} \frac{1}{|w|} \int \mathrm{d}P \frac{\sinh 2\pi P \sinh 2\pi P/k}{\cosh 2\pi P + \cos\pi(\frac{2\Delta}{w} - \frac{kw}{2})} \sum_{n=1}^\infty (-)^{an} n \ e^{-\frac{2\pi n}{\sqrt{2k}}\left(-\frac{kw}{2} + \frac{2\Delta}{w}\right)}$$

$$= \frac{1}{4k\prod_i R_i} \sum_{a,b} \sum_N \sum_{\tilde{\jmath}} \sin\frac{\pi(2\tilde{\jmath}+1)}{k}(-)^{bF} \times \tag{B.8}$$

$$\times D(N) \sum_{w\neq 0} \frac{1}{|w|} \int \mathrm{d}P \frac{\sinh 2\pi P \sinh 2\pi P/k}{\cosh 2\pi P + \cos\pi(\frac{kw}{2} - \frac{2\Delta}{w})} \frac{(-)^a}{\sinh^2\pi\left(\frac{\frac{kw}{2} + \frac{2\Delta}{w}}{\sqrt{2k}} + \frac{ia}{2}\right)},$$

which is the same as (4.13). The discrete representations contribution is obtained similarly.

C. Long strings emission for bosonic AdS$_3$ backgrounds

In this appendix we discuss the power-law corrections to t long string emission (4.13) in non-critical bosonic string backgrounds of the form $\mathrm{SL}(2,\mathbb{R})|_k \times \mathbb{R}^d$. In this case the counting of states can be done quite explicitly. Cancellation of the conformal anomaly in this background requires

$$k = 2 + \frac{6}{23 - d}. \tag{C.1}$$

Taking into account the differences between the superstring and bosonic string cases, the imaginary part of the annulus amplitude for long strings reads

$$\bar{\mathcal{N}}_c \ \sim \ \int \mathrm{d}^d p \ \int \mathrm{d}P \sum_N D(N) \sum_{w\neq 0} \frac{1}{|w|} \ \frac{\sinh 2\pi P \sinh\frac{2\pi P}{k-2}}{\cosh 2\pi P + \cos\pi(kw - E)} \frac{1}{\sinh^2\frac{\pi E}{\sqrt{k}}}, \tag{C.2}$$

with

$$E = \frac{kw}{2} + \frac{2}{w}\left[\frac{P^2 + 1/4}{k - 2} + \frac{\mathbf{p}^2}{2} + N - 1\right] \tag{C.3}$$

At high energies, this amplitude has the asymptotics

$$\bar{\mathcal{N}}_c \sim \sum_{w\neq 0} \frac{1}{|w|} e^{-\pi\sqrt{k}w} \left[\int \mathrm{d}P \ e^{\frac{2\pi P}{k-2} - \frac{4\pi}{w\sqrt{k}}\frac{P^2}{k-2}}\right] \left[\int \mathrm{d}p \ e^{-\frac{2\pi}{w\sqrt{k}}p^2}\right]^d \sum_N D(N) e^{-\frac{4\pi}{w}\sqrt{\frac{2}{k}}N}. \tag{C.4}$$

Let's first integrate over the $\mathbb{R}^d$ momentum (bringing a factor $w^{d/2}$) and over the AdS$_3$ radial momentum P:

$$\bar{\mathcal{N}}_c \sim \sum_{w=1}^{\infty} w^{\frac{d-1}{2}} e^{-2\pi\sqrt{k}\left(1-\frac{1}{4(k-2)}\right)w} \sum_N D(N) e^{-\frac{4\pi N}{w\sqrt{k}}} \tag{C.5}$$

The asymptotic density of states $D(N)$ corresponds to the counting of oscillator modes for $d+1$ bosons, in the light-cone gauge.

$$\frac{q^{(d+1)/24}}{\eta^{d+1}(\tau)} = \left(\prod_{n=1}^{\infty}(1-q^n)\right)^{-(d+1)} = \sum_N D(N) q^N . \tag{C.6}$$

One can use the generalized Ramanujan and Hardy formula:

$$D(N) \overset{N\gg 1}{\sim} \frac{1}{\sqrt{2}} \left(\frac{d+1}{24}\right)^{\frac{d+2}{4}} N^{-\frac{d}{4}-1} e^{2\pi\sqrt{\frac{(d+1)N}{6}}} . \tag{C.7}$$

Then one replaces the sum over N by an integral:

$$\sum_N N^{-\frac{d+4}{4}} e^{2\pi\sqrt{\frac{(d+1)N}{6}}} e^{-\frac{4\pi N}{w\sqrt{k}}} \sim \int \frac{dN}{N} N^{-d/4} e^{2\pi\sqrt{\frac{(d+1)N}{6}}-\frac{4\pi N}{w\sqrt{k}}} \tag{C.8}$$

$$\sim w^{-d/4} \int \frac{dx}{x^{1+d/2}} e^{-x^2+\sqrt{\frac{(d+1)\sqrt{k}\pi w}{6}}x} \sim w^{-\frac{d+1}{2}} e^{\pi\sqrt{k}\frac{d+1}{24}w}$$

Therefore using (C.1) all the exponential terms cancel. We remain with the power-law corrections, that don't depend on d:

$$\bar{\mathcal{N}}_c \sim \sum_{w=1}^{\infty} \frac{1}{w} \tag{C.9}$$

which is log-divergent. This demonstrates that all the non-critical bosonic AdS$_3$ backgrounds have a common divergence in long strings emission.

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